

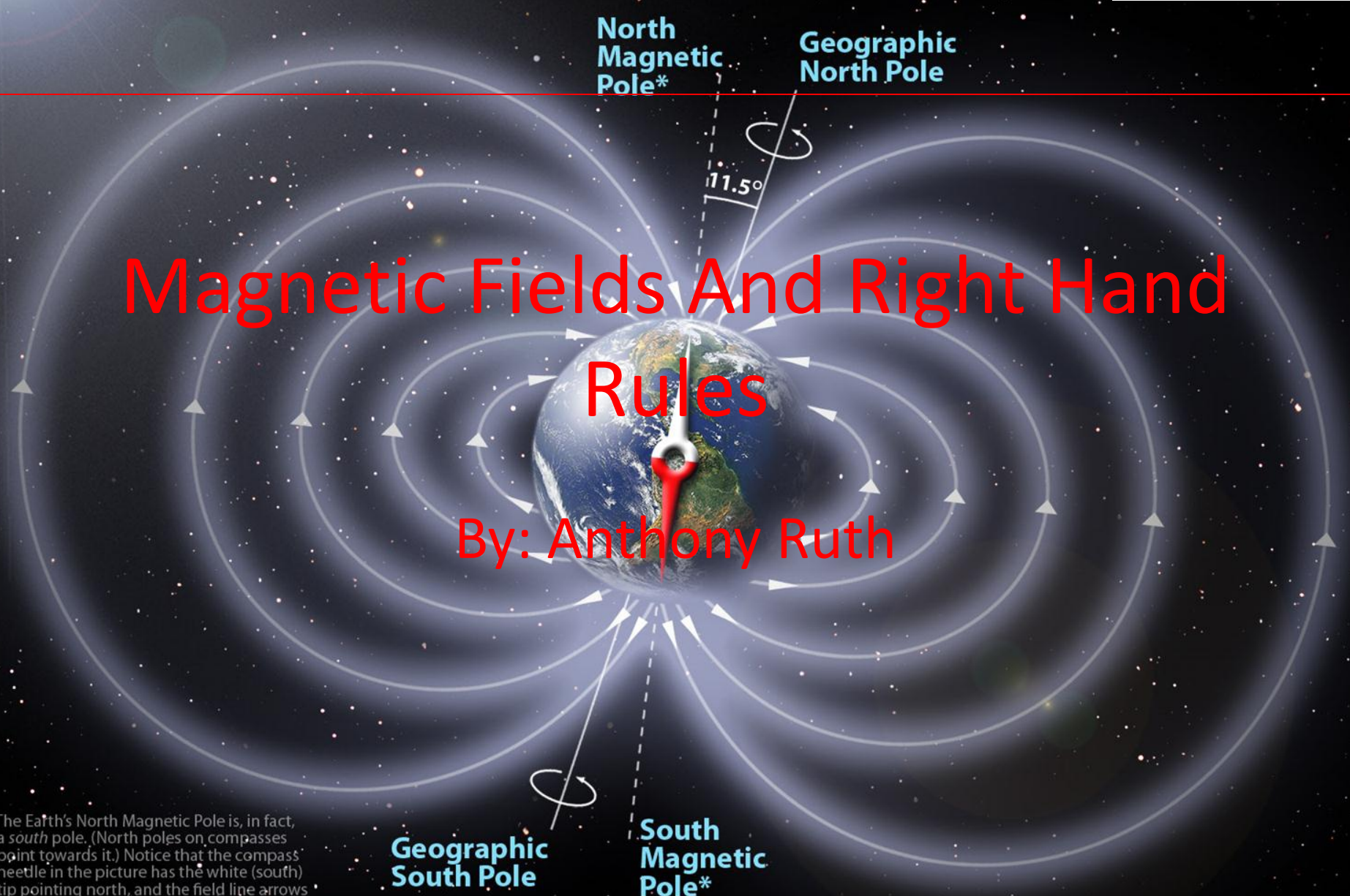
Magnetic Field & Right Hand Rule

Academic Resource Center

The Earth's Magnetic Field

Magnetic Fields And Right Hand Rules

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*The Earth's North Magnetic Pole is, in fact, a south pole. (North poles on compasses point towards it.) Notice that the compass needle in the picture has the white (south) tip pointing north, and the field line arrows point from south to north.

Magnetic Fields vs Electric Fields

- Magnetic fields are similar to electric fields, but they are produced only by moving charges while electric fields are produced by both moving charges and stationary charges.
- In addition, magnetic fields create a force only on moving charges.
- The direction the magnetic field produced by a moving charge is perpendicular to the direction of motion. The direction of the force due to a magnetic field is perpendicular to the direction of motion.

Magnetic Field Produced By a constant current

- The magnitude of a magnetic fields produced by a long straight wire with a constant current is given by

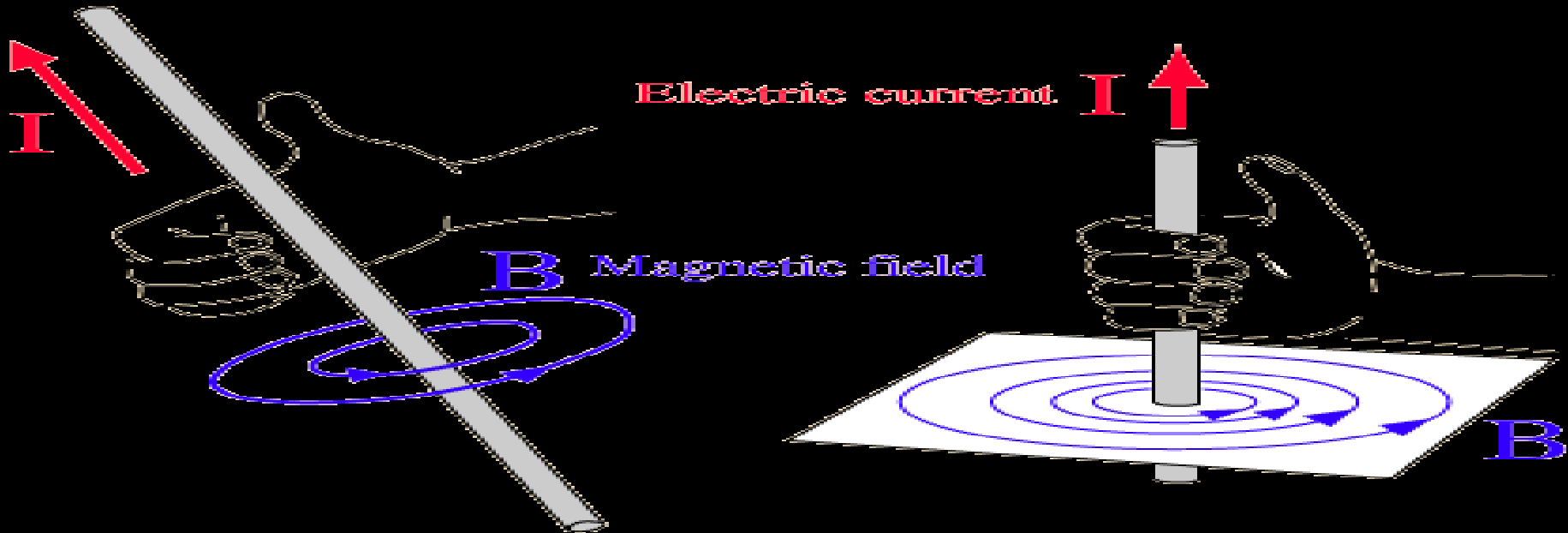
$$B = \frac{\mu_0 I}{2\pi r}$$

- Where B is the magnetic field, I is the current, r is the distance away from the wire, and μ_0 is called the permeability of free space.
- Magnetic fields are measured in Teslas(T). The Earth has a magnetic field of about $5e-5$ T.

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad \mu_0 = \frac{1}{c^2 \epsilon_0} = 1.26 * 10^{-6} \frac{Ns^2}{C^2}$$

Right Hand Rule for Magnetic Field Due to a Straight Wire

- To find the direction of the magnetic field use the right hand rule.
- Point thumb in direction of current
- The fingers will curl in the direction of the magnetic field



Vector Form of the Equation

- The magnitude and the direction of the magnetic field can be found using the vector form of the equation.

$$\mathbf{B} = \frac{\mu_0}{2\pi r^2} (\mathbf{I} \times \mathbf{r})$$

Force Due to a Magnetic Field on a Moving Charge

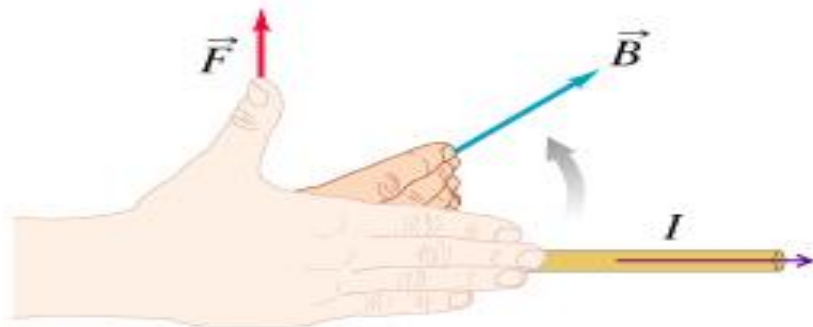
- The force exerted on a moving charge by a magnetic field is given by

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

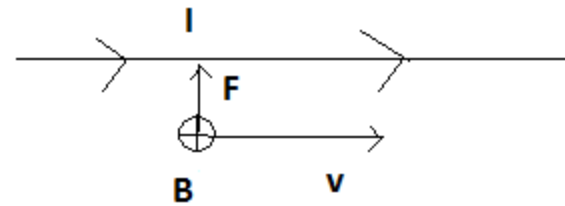
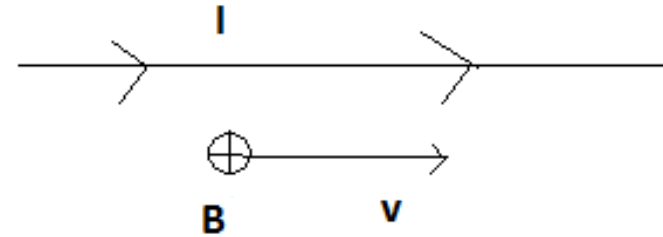
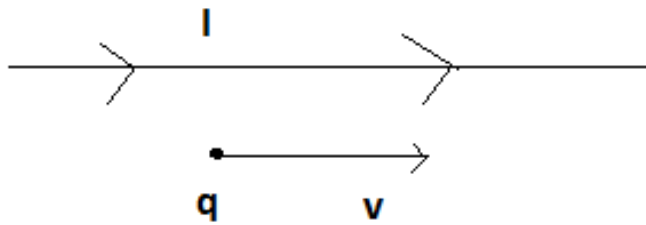
- Where F is the force vector, q is the charge of the moving particle, v is the velocity vector of the moving particle, and B is the magnetic field vector.

Right hand rule for force due to a magnetic field

- To find the direction of the force of a magnetic field:
- Point fingers in the direction of the velocity
- Curl fingers to the direction of the magnetic field
- Thumb points in the direction of the force



Example: Find the force on a moving particle due to the magnetic field of a wire



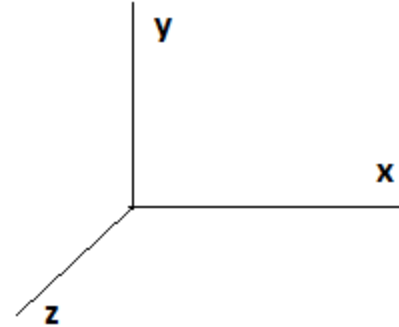
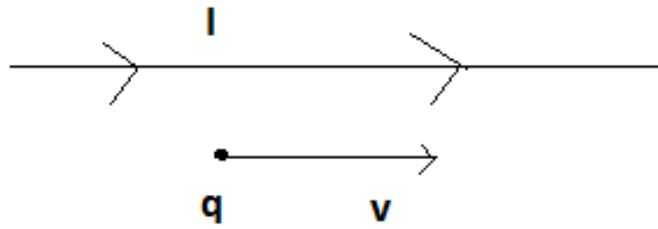
$$F = qv \times B = qvB \sin(a)$$

$$A = 90 \text{ degrees}$$

$$F = qvB = qv \frac{\mu_0 I}{2\pi r}$$

By the right hand rule the force points towards the wire.

Same problem using vector analysis



$$\mathbf{I} = (I, 0, 0) \quad \mathbf{v} = (v, 0, 0) \quad \mathbf{r} = (0, -r, 0)$$

$$\mathbf{B} = \frac{\mu_0}{2\pi r^2} (\mathbf{I} \times \mathbf{r}) = \frac{\mu_0}{2\pi r} (0, 0, -I)$$

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} = q v I \frac{\mu_0}{2\pi r} (0, 1, 0)$$

This is the same force as in the previous slide. Because it has a positive y -component, it points towards the wire.

Ampere's Law

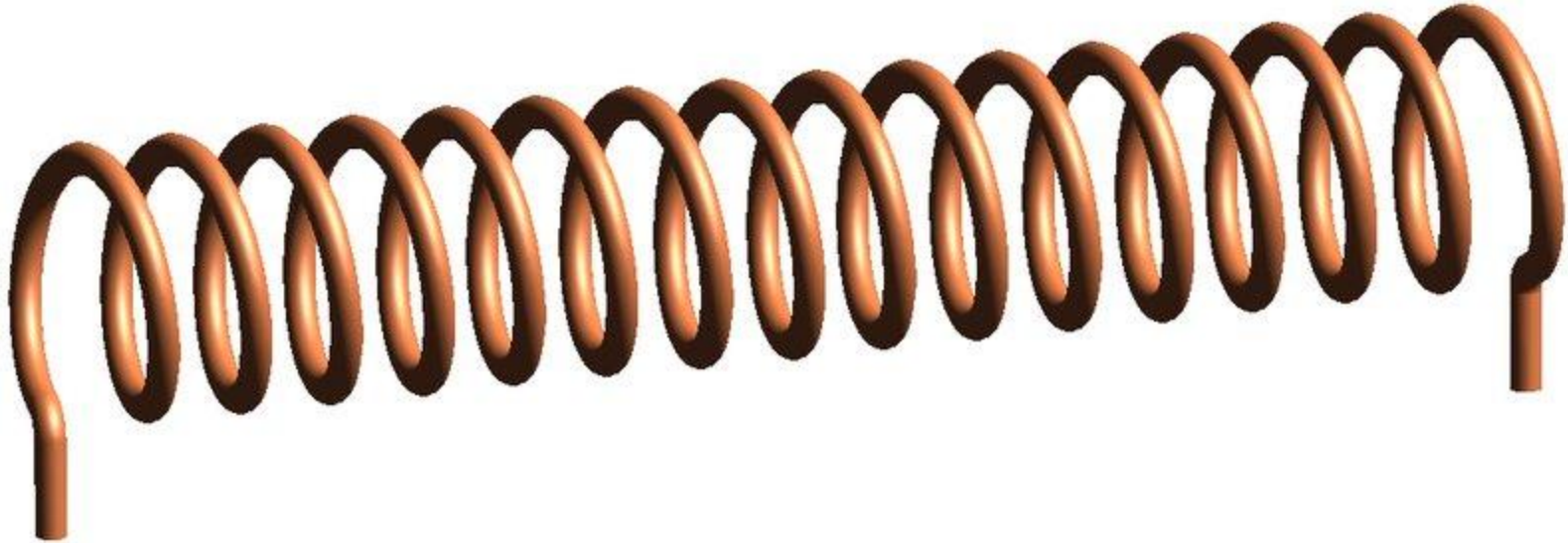
- Ampere's Law is very similar to Gauss' law. Gauss' law allows us to find the electric field on some surface that surrounds an electric charge. Ampere's law allows us to find the magnetic field on a closed loop that surrounds a current. In Gauss' law we want to choose our Gaussian surface so that the electric field is constant on the surface. In Ampere's law we want to choose our closed loop so that the magnetic field is constant on the loop. The form of Ampere's law for a loop with a constant magnetic field is:

$$\mu_0 I_{\text{inside}} = B \times P$$

- Where P is the perimeter of the loop. This equation is similar to Gauss' law for a surface with a constant electric field:

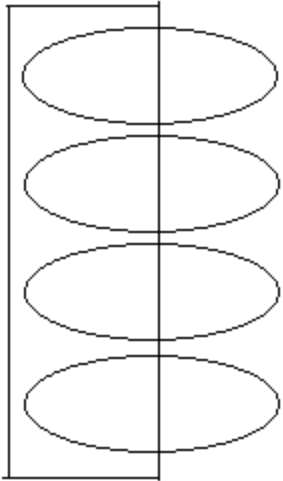
$$\frac{Q_{\text{inside}}}{\epsilon_0} = E \times A$$

Solenoids



A solenoid is many loops of wire with a current going through. Solenoids are used to generate magnetic fields. To find the magnetic field inside a solenoid we will make a simplified model. The model may differ a little from a real solenoid, but the agreement between the two is quite good. To calculate the magnetic field inside the solenoid we will remove the wires on the end, and treat the solenoid as infinitely many closely spaced rings. The spacing of the rings is given by $n = N/l$ the number of rings per unit length. The distance between two adjacent rings is $1/n$.

Simplified Model



To the left is a picture of the model we'll use to calculate the magnetic field inside a solenoid. Our model has infinitely many rings not just 4. The current going through the loop is j * the current going through each loop. The distance between each loop is $1/n = l/N$ so the perimeter of the loop for Amperes Law is $2R + j/n$. Therefore using amperes law the magnetic field in the loop is:

$$\lim_{j \rightarrow \infty} \frac{\mu_0 I (2R + j)}{\frac{1}{n} * j} = \mu_0 I n$$

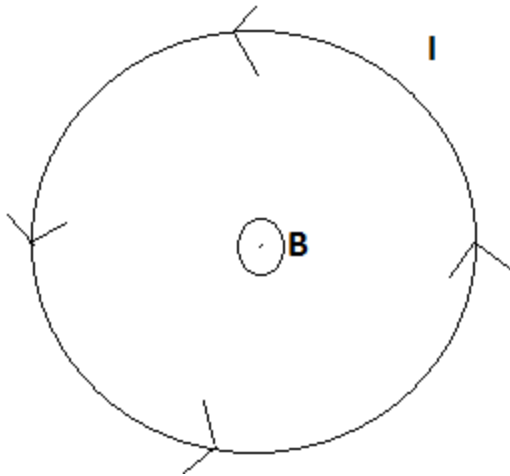
For an alternative derivation see Physics for Scientists and Engineers Chapter 29.

Biot-Savart Law

- Wires aren't always straight. We need a method to calculate the magnetic field regardless of the shape of the wire. To do this we use the Biot-Savart Law. The Biot-Savart Law is

$$dB = \frac{\mu_0}{4\pi} \frac{I ds \times r}{r^3}$$

Magnetic Field in the Center of a Current Loop



We have a current, I , going counter-clockwise around in a closed loop. From the right hand rule we can see that in the center of the loop the magnetic field points out of the page. Using the Biot-Savart law

$$\mathbf{B} = \int d\mathbf{B} = \int \frac{\mu_0}{4\pi} \frac{I ds \sin\theta}{r^2} = \frac{\mu_0}{4\pi} \frac{I}{R^2} \int ds = \frac{\mu_0 I}{2R}$$

The integral over the ring is $2\pi R$.