# Relationship between Fourier Space and Image Space

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# Presentation Outline

- What is an image?
- Noise
- Why do we transform images?
- What is the Fourier Transform?
- Examples of images in Fourier Space
- Image composition in Fourier Space
- How to interpret Fourier Space?
- Filtering
- Review

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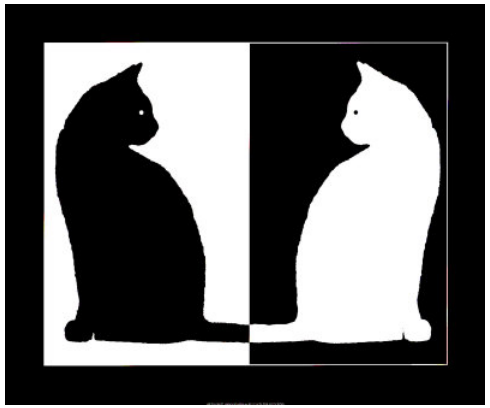
# What is an image?

- For this workshop, an image is a two dimensional matrix with values that specify its intensity
- There are different kinds of images
  - Black and White (values of 1s and 0s)
  - Grayscale (typically with values between 255 and 0)
  - Color (usually split between 3 matrices with each matrix for a different color, RGB)



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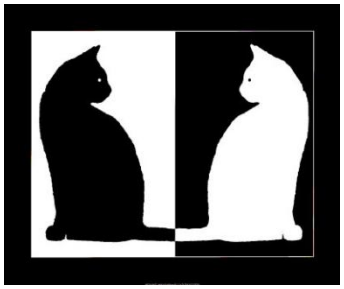
# What is an image?

- **IMPORTANT: IT MUST BE UNDERSTOOD**
  An image is represented as a two dimensional matrix with values corresponding to intensity
- This means we can view images like so:



$$\begin{bmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \cdots & 255 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 255 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 255 \\ \vdots & \ddots & \vdots \\ 160 & \cdots & 40 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \cdots & 255 \\ \vdots & \ddots & \vdots \\ 160 & \cdots & 40 \end{bmatrix}$$

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# Noise

- Can be random or predictable
- Different categories
  - Gaussian
  - Salt and pepper
  - Motion blur
  - Etc.

Gaussian

Salt and Pepper

Mot

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# Noise

- Caused by problems with data acquisition
- Can be removed by accounting for them during data acquisition
  - However, most of the time, the image is already constructed and data cannot be taken again
- In this case, it is necessary to use filters
  - There is a preferred filter for each type of noise
  - Filters will be discussed later in the workshop

# Why do we transform images?

- Images can be analyzed in different kinds of spaces
- The purpose is not to complicate the information but change the way we view the information
  - For example, two can be represented as 1+1, 2cos(0), 2sin(pi/2), 2*1, sqrt(4)
- There are various types of transformations
  - Discrete Cosine Transform, Fourier Transform, Discrete Wavelet transform, and etc.
- <u>There is more than one way to see an image!</u>

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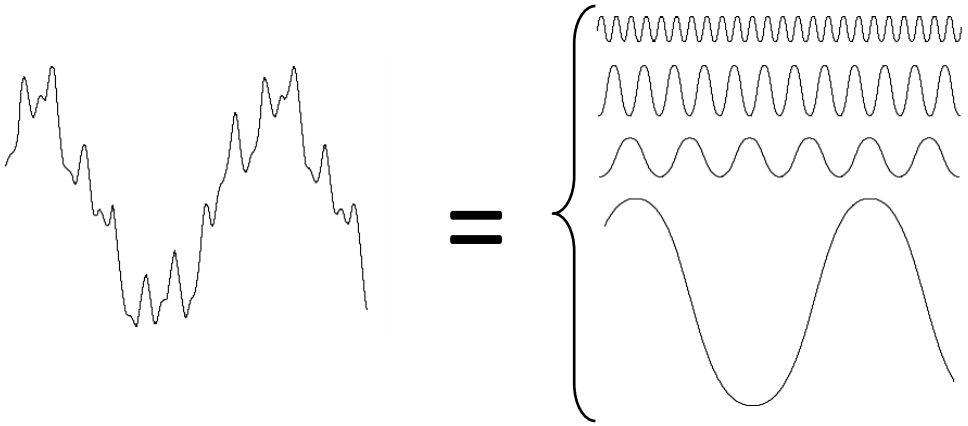
# What is the Fourier Transform?

- The main idea of the Fourier transform is that a complex signal can be expressed as the sum of sines and cosines of different amplitudes



- The Fourier transform translates the image as frequency data
- The equation for a 2-D Fourier Transform is:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

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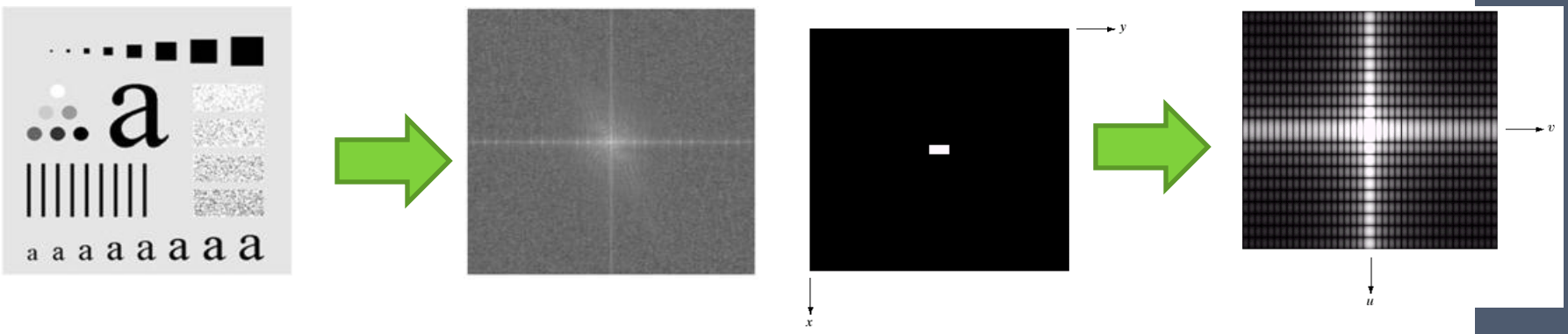
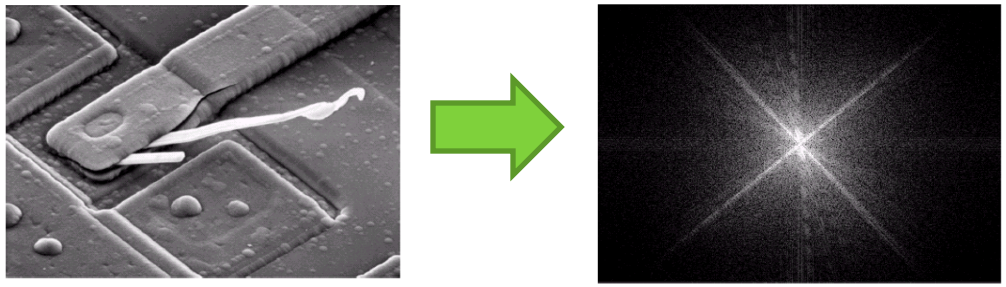
# Examples of Images in Fourier Space

- For each image, the Fourier spectra is displayed



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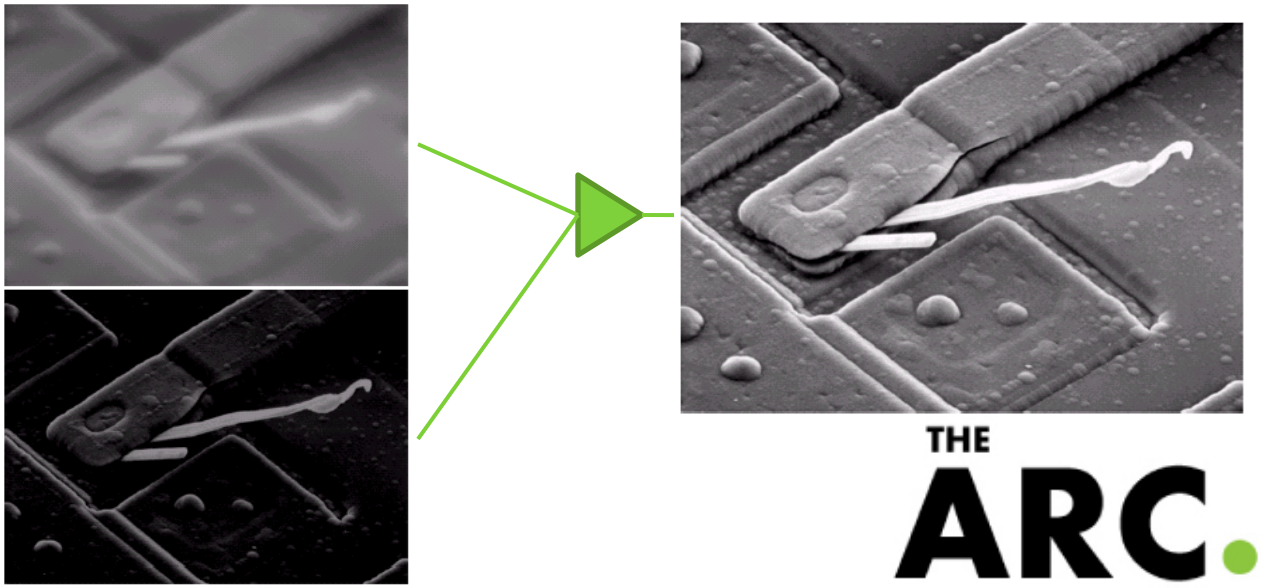
# Image composition in Fourier Space

- An image can be represented as two components: high frequencies and low frequencies
  - Low frequencies make up the bulk of the information (areas of low variation in intensity)
  - High frequencies make up the edges and fine detail (areas of high variation in intensity)

Low Frequencies only:

High Frequencies only:



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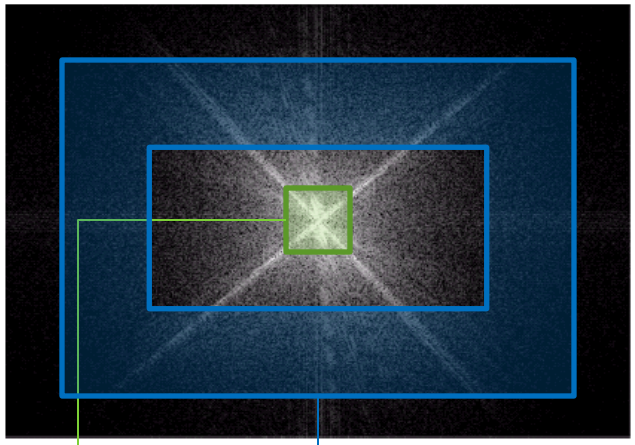
# How to interpret Fourier Space?

- The Fourier Spectra shows both low and high frequency components
  - Low frequencies are near the origin
  - High frequencies are away from the origin

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| | | | | | |
|---|---|---|---|---|---|
| 0 | 3 | 6 | 8 | 1 | 1 |
| 1 | 2 | 12 | 14 | 3 | 2 |
| 2 | 1 | 15 | 20 | 2 | 1 |
| 4 | 4 | 13 | 25 | 1 | 1 |
| 1 | 2 | 11 | 10 | 0 | 2 |
| 2 | 3 | 3 | 5 | 5 | 1 |

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# Filtering

- The purpose is to modify the image to either remove noise, emphasize, and/or de-emphasize certain components
- Filtering can be done in both Fourier Space and Image Space
  - Filtering in Image Space uses convolution
  - Filtering in Fourier Space uses multiplication

- In Fourier Space, filtering is implemented by multiplying the image's Fourier spectra, F(u,v), with the filter spectra, H(u,v)
  - $F(u,v) \times H(u,v) = Y(u,v)$
- The filter spectra, H(u,v), is carefully designed to fit the application

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# Filtering

- Like mentioned before, there are many types of filters:
  - Median filtering
  - Average filtering
  - Low pass filtering
  - High pass filtering
  - Max filtering
  - Min filtering

- The most commonly used ones are ‘Low Pass’ and ‘High Pass’
- Knowledge of the type of noise affects filter choice

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# Review

- An image is represented as a two dimensional matrix with values corresponding to intensity
- Noise can be random or predictable
  - There are different categories for noise
- Transformations allow us to look at images in a different light
  - The purpose is not to complicate the information but change the way we view the information
- The main idea of the Fourier transform is that a complex signal can be expressed as the sum of sines and cosines of different amplitudes
- An image can be represented as two components: high frequencies and low frequencies
- The purpose of filtering is to modify the image to either remove noise, emphasize, and/or de-emphasize certain components

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# References


- Main Source: http://www.comp.dit.ie/bmacnamee Graphics and Image Processing Powerpoint
- Wikipedia
- BME330
- BME438
- ECE507


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