

Center of Mass

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Center of Mass

- Center of Mass is defined by the 3rd Edition Ohanian as the average position of the mass of the system.
- Mathematically, it is defined by the following integral:

$$x_{CM} = \frac{1}{M} \int x \, dm \quad (1)$$

- For defined points of mass in a system, this integral can also be written as the following sum:

$$x_{CM} = \frac{1}{M} \sum_{i=1}^n m_i x_i \quad (2)$$

- In other words, the center of mass is sum of the mass fraction of each point in the system multiplied by its position. In this case M is the total mass of the system.
- The previous equations describe the position of the center of mass in the x direction, but the same equations apply for the y and z directions as well.

Motion of the Center of Mass

- The velocity and acceleration of the center of mass of a system is found the same way as the center of mass:

$$v_{CM} = \frac{m_1 v_1 + m_2 v_2 + \dots + m_n v_n}{M} \quad (3)$$

$$a_{CM} = \frac{m_1 a_1 + m_2 a_2 + \dots + m_n a_n}{M} \quad (4)$$

- The advantage to using the center of mass to evaluate the motion of a system is that the center of mass acts the same as a single particle:

$$P = M v_{CM} \quad (5)$$

$$F = M a_{CM} \quad (6)$$

Example Problem 1

A 59-kg woman and a 71-kg man sit on a seesaw, 3.5 m long. Where is their center of mass?

Example Problem 1 (Solution)

This problem is done very easily using equation 2 with two points: the man and woman. Each point has a mass and position, and it is merely a matter of defining the position in terms of some arbitrary origin, and inputting the values into equation 2. This will output an x position relative to the origin that has been decided on.

In the case of this solution I will use the woman as the origin.

Example Problem 1 (Work)

$$m_{\text{woman}} = 59 \text{ kg}$$

$$m_{\text{man}} = 73 \text{ kg}$$

$$x_{\text{woman}} = 0 \text{ m}$$

$$x_{\text{man}} = 3.5 \text{ m}$$

$$x_{CM} = \frac{(59 \text{ kg})(0 \text{ m}) + (73 \text{ kg})(3.5 \text{ m})}{132 \text{ kg}}$$

$$x_{CM} = 1.94 \text{ m}$$

Example Problem 2

Mount Fuji has approximately the shape of a cone. The half-angle at the apex of this cone is 65 degrees, and the height of the apex is 3800 m. At what height is the center of mass? Assume that the material in Mount Fuji has uniform density.

Example Problem 2 (Solution)

This problem is far more complex than the previous problem and will involve equation 1. We need to begin by defining terms in the equation in terms of values that we are given. To begin with, the problem should be set up like example 6 in the Chapter 10 of the Ohanian Physics book, so that the cone has the y axis point downward from the vertex. We can then define the radius of the cone in terms of its distance in the y direction using the angle we are given. We can define the cone in terms of circular slabs emanating downward from the tip with a thickness of dy and a mass of dm . dm can be defined in terms of y and dy with substitution and using ρ as the density. Once all the terms are defined, the integral can be taken from $y=0$ to $y=h$.

Example Problem 2 (Work)

$$\phi = 65^\circ$$

$$r = y * \tan(\phi)$$

$$dm = \rho(\pi r^2)dy$$

$$dm = \rho(\pi y^2 \tan^2(\phi))dy$$

$$M = \int_0^h dm$$

$$y_{CM} = \frac{\int_0^h y dm}{\int_0^h dm}$$

$$y_{CM} = \frac{\int_0^h y * \rho(\pi y^2 \tan^2(\phi))dy}{\int_0^h \rho(\pi y^2 \tan^2(\phi))dy}$$

$$y_{CM} = \frac{\int_0^h y^3 dy}{\int_0^h y^2 dy}$$

$$y_{CM} = \frac{3y^4}{4y^3} \Big|_0^h$$

$$y_{CM} = \frac{3h}{4} \text{ (measured from top)}$$

$$h = 3800 \text{ m}$$

$$y_{CM} = 950 \text{ m above base}$$

Example Problem 3

A fisherman in a boat catches a great white shark with a harpoon. The shark struggles for a while and then becomes limp when at a distance of 300 m from the boat. The fisherman pulls the shark by the rope attached to the harpoon. During this operation, the boat (initially at rest) moves 45 m in the direction of the shark. The mass of the boat is 5400 kg. What is the mass of the shark? Pretend that the water exerts no friction.

Example Problem 3 (Solution)

This problem is very simple to solve because friction is ignored. This means there are no external forces, so the center of mass of the system (the shark and boat) does not move at all. With this knowledge, it is a simple matter of defining the terms of equation two and then solving for the mass of the shark. The best point to use as an origin in a problem like this is the center of mass, because it will not be moving. Both the shark and the boat will end with their positions on the center of mass.

Example Problem 3 (Work)

$$x_{boat} = -45 \text{ m} \quad x_{shark} = 255 \text{ m} \quad m_{boat} = 5400 \text{ kg}$$

$$x_{CM} = \frac{x_{shark}m_{shark} + x_{boat}m_{boat}}{m_{shark} + m_{boat}}$$

$$x_{shark}m_{shark} + x_{boat}m_{boat} = 0$$

$$m_{shark} = \frac{-x_{boat}m_{boat}}{x_{shark}}$$

$$m_{shark} = \frac{(45 \text{ m})(5400 \text{ kg})}{(255 \text{ m})}$$

$$m_{shark} = 953 \text{ kg}$$