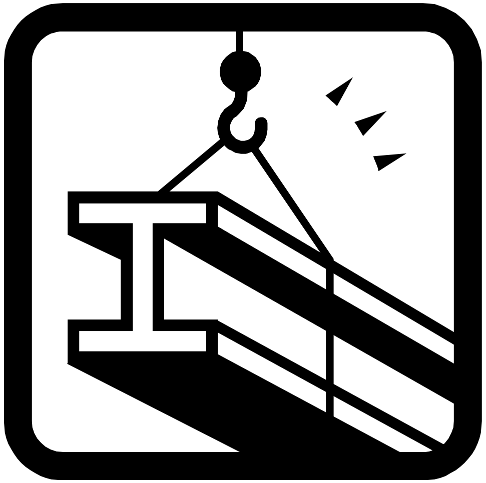
Bending

for MMAE and CAE students



Academic Resource Center

Topics Covered

1) General analysis of forces in beams
2) Determination of stresses in beams
    1) Compressive and tensile stresses
    2) Shear stresses
3) Bending of composite materials
4) Notes on design
5) Questions/practice problems

Analysis of Forces in Beams

Notes:

[SHEAR, UNITS OF "FORCE"]

$$V_i(x_i) := V_0 + \int_0^{x_i} w(x_i)\,dx_i$$

[BENDING, UNITS OF "FORCE*LENGTH"]

$$M_i(x_i) := M_0 + \int_0^{x_i} V_i(x_i)\,dx_i$$

- t
- es
- e”

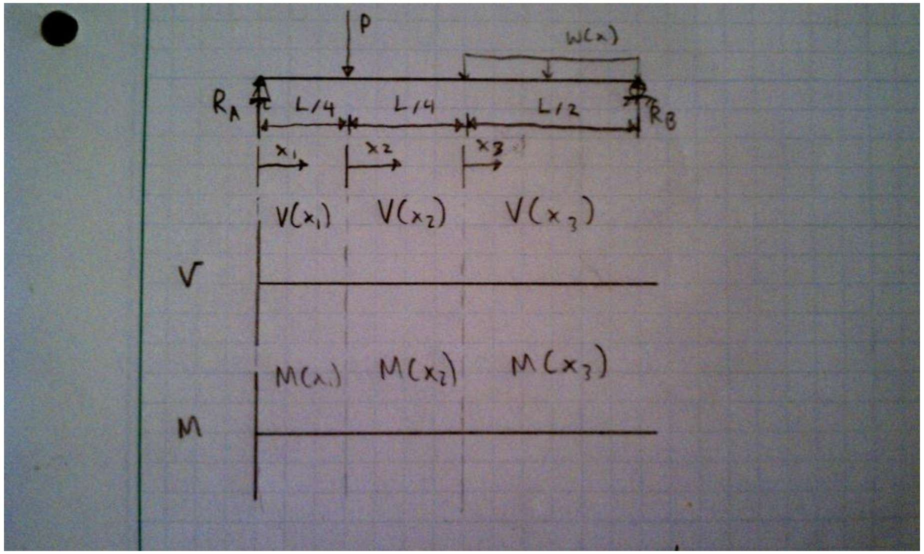
  of function at x.i=0 of evaluation
- X.i is “x axis” for a continuous function on the beam

W(x) is distributed load with units “force/length”

Analysis of Forces in Beams

Notes:

- Notice zones x.1, x.2, x.3 at changes in loading
- Notice functions of V and M for each different zone
- Reactions R.A and R.B are solved from equilibrium at start of problem



Determination of Stresses in Beams

- Based on assumptions:
  - Planes remain plane and normal
  - Small, linear deformations
  - Stresses within elastic range of materials (therefore governed by Hooke's Law)
- Second assumption is checked by:

RADIUS OF CURVATURE

$$\rho := \frac{E \cdot I}{M}$$

WHERE:
E: MODULUS OF ELASTICITY
I: 2nd MOMENT OF INERTIA
M: BENDING MOMENT
ρ: RADIUS OF CURVATURE

CHECK FOR SMALL DEFORMATIONS

$$\rho > 10 \cdot h$$

WHERE:
ρ: RADIUS OF CURVATURE
h: HEIGHT OF MEMBER

Determination of Stresses in Beams

Notes:

- “y” is taken from neutral axis
  - This convention will make points at the bottom of the beam “-y” negative, which indicates tension and vice versa for compression
- “I” is relative to neutral axis
  - Utilize parallel axis theorem shown in equation 3!!!
- Equation 1 applies for all cross sections
- Equation 2 applies for prismatic (rectangular) cross sections

AXIAL STRESS σ.x AT ANY POINT "y" ON BEAM CROSS SECTION:

$$\sigma_x := \frac{M \cdot y}{I}$$

WHERE: y: VERTICAL COORD. OF INTEREST ON BEAM
I: SECOND MOMENT OF INERTIA

$$I := \int y^2 \, dA \qquad \text{EQ 1}$$

$$I := \frac{b \cdot h^3}{12} \qquad \text{EQ 2}$$

$$I_{cent} := I_0 + A \cdot d^2 \qquad \text{EQ 3}$$

Determination of Stresses in Beams

Maximum Axial Stress

- "c" is the position of the centroid relative to the top or bottom of the member
- From previous relationship, if y=c, stress is evaluated at a maximum point

$$\sigma_{max} := \frac{M \cdot c}{I}$$

THEREFORE

$$\sigma_{max} := \frac{M}{S}$$

WHERE THE SECTION MODULUS, S, EQUALS:

$$S := \frac{I}{c}$$

Determination of Stresses in Beams

Vertical Shear Stress

- A shear force "V" causes a non uniform distribution of shear stress on a member's cross section
- Approximations to the right apply ONLY for rectangular cross sections

$$\tau_{AVG} := \frac{V \cdot Q}{I \cdot t}$$

WHERE: t:THICKNESS OF MEMBER PARALLEL WITH NEUTRAL AXIS AT POINT OF INTEREST

$$\tau_{MAX} := \frac{3 \cdot V}{2 \cdot A}$$

WHERE: A:CROSS SECTIONAL AREA OF MEMBER

Determination of Stresses in Beams

Shear Flow

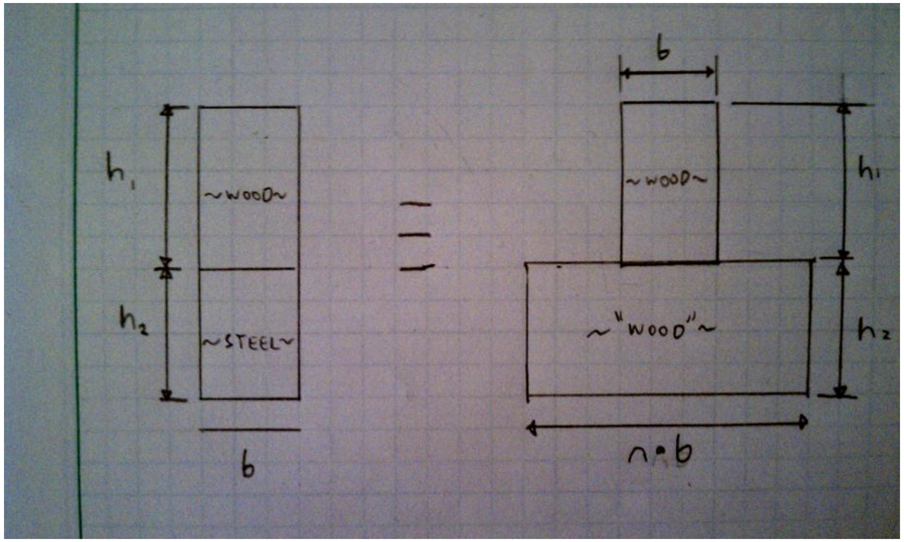
- As a member undergoes bending, portions of the cross section try to “slide past” one another
- The shear causing this sliding is known as shear flow “q”
- “q” is a “force/length” on an axial face of the member (perpendicular to cross section)

$$q := \frac{V \cdot Q}{I}$$

WHERE:

V: SHEAR FORCE PARALLEL TO CROSS SECTION
Q: FIRST MOMENT OF INERTIA ABOUT CENTROID
I: SECOND MOMENT OF INERTIA ABOUT CENTROID

Bending of Composite Members



Transformed Sections:

- For cross sections made of two materials
- "n" is relationship between the modulus of elasticity of the two materials
- Notice constant height of areas of cross section
  - Second moment of inertia and centroid are calculated using "n*b"

Notes on Design

1. Start by determining shear and bending moment diagrams!
   - Do not forget the member's self weight (which becomes a distributed load)
   - It is advantageous to separate different loading types and develop a shear and bending moment diagram for each (different "load factors" apply in design)
2. "S" the section modulus of a member is a commonly available property in documents such as the American Institute of Steel Construction's Steel Manual
3. The maximum allowable stress of a material is a commonly available property for steel, concrete, etc.

Practice Problems

1. 5.76 page 339 of reference [analysis for shear and bending]
2. 4.9 page 225 of reference [analysis for shear and bending, determination of axial stresses]
3. 6.1 page 384 of reference [determination and usage of shear flow]
4. 6.5 page 386 of reference [analysis for shear and bending, design of maximum shear and axial stresses]
5. 4.50 page 240 of reference [stresses in composite cross sections]

References

Beer, Ferdinand et. al. *Mechanics of Materials,* 4TH Edition. 2006. McGraw Hill.