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Scientist, Colleague, Mentor, and Father
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Those who inspired and encouraged us are never gone; they are in our minds and our hearts forever. Howard Brenner is in this group. The research ideas of many of us originated from reading Howard’s work or talking with him. He was a pure scholar with high principles and a self-deprecating humor that allowed us to remain friends even in disagreements. He was incredibly disciplined and often rigid, yet so human and warm.

Howard Brenner’s scientific impact went far beyond fluid dynamics. His work is quoted through the literature on interfacial science, colloid dynamics, bioengineering, and heterogeneous catalysis. He was a true intellectual leader who positively affected the careers of many.

On November 16, 2014, the American Institute of Chemical Engineers sponsored a technical symposium in honor of Howard Brenner’s work and life. This book is based on those presentations. Many of us felt that a more permanent remembrance of Howard should be created—something to which future engineers and scientists could refer to learn more about the man. I thank the authors who contributed to this volume for expanding upon their presentations and for adding anecdotes and personal recollections that properly reflect the human side of this great scholar. Surely there are even more interesting stories about Howard, the person and academic, than covered in this tribute, but at least we have recorded some memories about the person who influenced us as a scholar, colleague, mentor, and father.

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John Anderson’s inviting me to speak about my dad as a father and grandfather provided a wonderful opportunity for me to reflect about a different side of my father, and to spend time walking down memory lane both smiling and teary-eyed.

At other memorials, I talked mostly about my dad’s intellectual life. Frankly, when I reflected on his life, it was his passion for a world of scientific ideas that most stood out. From the time I was very young, I associated my father with his books and with his solitary, late nights working on his theories. My contribution then was to make him coffee following the catnap he often took after dinner, so he could stay up ‘til the wee hours of the morning lost in his work. He loved having me or my sisters, or my mom doting on him. All women in the house—and him. What’s not to love. I’m actually convinced, by the way, that not only did we do all the things so many women did in the 50s and early 60s to make life easier for men—but in my father’s case—I believe we actually helped him in his scientific quest to understand fluid flow. He was fascinated with the viscosity of a bubble-infused hair gel I used called Dippity Do, which helped tame my wavy hair. And, after all, the theories in Low Reynolds Number Hydrodynamics had to come from somewhere…

When I saw the space film Interstellar recently, I found myself teary-eyed at the father-daughter relationship central to the film, but also at my recollections of my dad waking me in the middle of the night during many a lunar eclipse to share the moment with me, using pieces of fruit to demonstrate how the eclipse worked. An apple, an orange, and the banana that so well represented the moon’s crescent were our props. Or, sitting together in the early 60s, watching the space-age unfold before our eyes, with the trusting voice of newsman Walter Cronkite narrating as man reached for the stars. My sister, Joyce, recalls Dad regularly using the Twilight Zone TV series to offer her scientific explanations for the freakish sci-fi storylines. He loved finding science in everyday life, and sharing it with his family.
John asked me to talk about my dad’s ‘legendary’ sense of humor. My father had a stubbornly serious side and a tremendous sense of drive and purpose—but he also possessed a wicked, sometimes sophomoric, sense of humor… For years, my dad found it quite amusing to steal my dinner plate if I stepped out of the room. He would simply be resting the plate on his lap, out of sight, but he never ceased to find this extraordinarily amusing. And one day, when a bee flew into our house, both out of an abundance of caution, and to add levity to the situation, he donned what amounted to a full suit of armor. I can’t recall the exact garb, but I believe it involved a winter face mask, with his glasses placed over it, thick gloves, and other protective gear. Armed with a fly swatter, he dramatically locked himself in the bathroom, where we heard all sorts of yelling from behind the door as he worked to vanquish this poor little bee. About 10 minutes later, he emerged the victor, still dressed in his full regalia. He was an enormous tease, and while we didn’t always see the humor, we accepted that was just who Dad was. By the way, the night before my husband and I married, my dad left Steve a note letting him know it was not too late to change his mind! Such a tease! I believe strongly that Dad used humor initially to cope with the insecurity of having grown up in an uneducated family so he could feel comfortable in a scholarly world by disarming the people around him. Later, as he became the scholar, it helped others realize that he was just an ordinary guy like they were. And, indeed, that’s exactly how he felt despite his many academic achievements and illustrious awards.

Another side of my dad you might not have expected in a man so focused on the logical and rational was that he was remarkably sentimental. Going through his papers, I found calendar date books filled with dozens of memories of the important people in his life—their birthdays, anniversaries, and sometimes dates of passing away… He related to me more times than I can remember how traumatized he was when, as a young father, he was outside on a cold day washing his car, when I woke from my nap, emerged in pjs and no coat, to see what he was up to. The story goes that I had a fever, and seeing me standing there in the cold, broke his heart into a thousand pieces because he felt he hadn’t lived up to his responsibility. He swooped me up and carried me back into our warm house. I can tell you I was still hearing this story even after I had turned 60 just a couple of years ago!

And my dad talked a great deal about his own father, Max Brenner, who left school after the 8th grade, and was an undeniably colorful character. Ever the entrepreneur, but also a bootlegger, some of his projects drew my dad to science, including his early forays into silver polishing formulas.

My dad also shared many times how I’d helped him stay awake on the long trip back from the Catskills after my great grandmother passed away. Just Dad and I made this journey to pack up our summer bungalow, and he was completely exhausted by the time we headed home in the wee hours of the morning. Though only 10, I was keenly aware of just how bone-weary he was, and I kept him talking the entire ride back to be sure he could safely navigate the winding mountain roads. He never forgot this trip, and shared it with me regularly. These were among the sentimental memories he stored in between the thousands of pieces of arcane data and differential equations that occupied most of his brain. But they were so important to him because of his great love of his family. And though I can’t recall exactly what we talked about that night, I suspect they included the same themes echoed by my father throughout my life. He was a man of principle who often championed the underdog. He believed in social justice and equality of opportunity, and he made sure to communicate those values to his children. He had little use for material things, but was generous to
a fault in making sure the people in his life had what they needed. He thought of himself as an everyday Joe, and having never lost his Bronx accent, he sounded very much the part.

Then there was my dad, the homework helper. Suffice it to say that I believe our homework interchanges were equally frustrating to both of us—especially when it came to math! I can tell you without fear of contradiction that my sisters and I inherited our mother’s sense of math. I was good at algebra, but my nemesis was solid geometry. When I wanted my dad’s help, I begged him to simply give me the formula to solve the problem! I didn’t want explanations involving Euclidian logic or Pythagorian theorem. He, on the other hand, wanted to espouse on exactly those things, and he regularly insisted I inform my teacher why he or she was teaching the course entirely wrong! So here I was with what I believed to be a mathematical genius for a father, yet I never received any easy or direct answers to my questions! He, on the other hand, kept longing for at least one of his three daughters to embrace his love of mathematical exploration…. Alas, neither was meant to be…

Actually, my favorite stories about my dad’s involvement with my scholarly life involve two science fair projects. The first took place when I was 10, using a Worldbook Encyclopedia entry to build a simple motor. The instructions seemed easy enough, and my dad dutifully helped me collect the wire, dowel and other items I needed, but my motor didn’t work. My dad was as frustrated as I was, and decided to take my creation to NYU, enlisting his Chemical Engineering colleagues to evaluate what went wrong. When he returned home, I was both elated and dumbfounded. My motor had entirely new materials, and looked nothing like it had when he left that morning. Dad proudly showed me what his colleagues had reconfigured, and he switched it on. The motor turned exactly one rotation and promptly died, never to work again!

My other science project drew Dad into a life of larceny. But it was for the love of his daughter that he couldn’t bear to let me fail. And so, suffice it to say that the photos I mounted on a poster to show the effects of organic versus inorganic fertilizers on plant growth were not exactly as they appeared to be. As best I can recall, the ‘before’ pictures were taken after the fact, and the ‘after’ pictures were duplicitously taken to demonstrate my so-called scientific findings. Dad was a forgiving guy, who didn’t want his child to be crushed, but who nevertheless didn’t hesitate to impart a life’s lesson then, and often, throughout my life. So many of you—his colleagues and former students, probably recognized this attribute again and again in your own interchanges with him as his way of encouraging you to dig deeper and try harder.

As for Dad’s seven grandchildren, at last, at least one, Max, now almost 17, seems destined for a life in science, or perhaps his younger brother, Miles! Our fingers are crossed! My son, Alex, who in many ways reminds me of my dad temperament-wise—and that is both the good and the bad news(!)—spent several months helping care for him in the last year of his life. Alex spoke beautifully at the Boston memorial, observing that... “As anyone who knew [my grandfather] Howard ... could attest, he could no doubt inspire and infuriate in equal measure. He was at times a difficult man, one who would often push and prod people beyond reasonable measure... Alex added, “As I got older, I gradually learned that it was this same drive, profound intellectual curiosity and sense of perfectionism that made him so valuable not only to the scientific community, but to his family as well. He endowed each of his children and grandchildren with an incredible thirst for knowledge, perhaps along with a certain portion of his incorrigible nature and pointed humor, and for that we’re all deeply indebted to him.”
My daughter, Margo, echoed Alex's sentiments, offering that Papa Howie could get under your skin with his comments, "exposing a nerve or being more than a bit annoying if he thought it would lead to some kind of intellectual or other enlightenment. He loved nothing more than psychoanalyzing those around him, having himself undergone the process for years. Margo observed that this prodding helped create personal insights in each of us, that while sometimes not especially welcome, were frequently dead-on. And I think there's no doubt that this is exactly what he did with his students and sometimes, colleagues—in those instances, bringing intellectual enlightenment, or at least a thirst to seek out such enlightenment. We didn't always love him for thrusting us into the center of this particular universe, but we recognized its enormous value.

At the end of the day, Howard Brenner, my father, was, fundamentally a simple guy from a humble background, who happened to possess extraordinary intellectual gifts and insights, a dogged work ethic, and a mischievous sense of humor. An imperfect man who, nevertheless, wanted to make the world a little more perfect through his commitment to pursuing scientific truths—inspiring his young protégés and colleagues alike. And letting his children know they could be anything they set their minds to be.

In closing, I want to thank you all for paying tribute to my dad. My family hopes that some of you, or others you might encourage, will follow up on his final efforts to delve into Navier-Stokes. Not only did he talk about this work for so many years, but I was witness just four nights before he passed away to his making the final revisions on what was to be his final paper. At that point in his illness, even the slightest exertion was an arduous encounter in which he fought to continue breathing—but there was a remarkably peaceful state that emerged as soon as he could catch his breath, reposition his oxygen and begin to type on the computer. So, while I'm aware that some of his colleagues will eventually be running simulations and such on his last work, I hope there will be more. Dad confided that some may have thought he had gone off the deep end in his later years, or been overly preoccupied with a tangential aspect of these equations, but I know his fervent hope was that his ideas would, at the very least, be thoroughly explored. And so, I ask you today to take this challenge seriously if you're able to do so as a way to honor his achievements, his scientific passion, and help perpetuate his legacy.

A final request—that one or more of you create a Howard Brenner Wikipedia entry. There's an abbreviated German Wikipedia posting, but nothing in English. I would be glad to edit the personal part of the entry, and to help in any way to see this accomplished.

On behalf of our family, thank you for allowing me to share these special memories, and thank you for honoring my father by your presence here today.
During his long (60+ years) professional career, Howard Brenner made an astonishingly large number of seminal contributions to a variety of topics in fluid mechanics such as, to name but a few: particle motions in very viscous fluids, the mechanics of complex fluids, multiphase flow in porous media, emulsion rheology and many others. In my talk I shall focus on a few of his early publications in “low-Reynolds number fluid mechanics” which helped transform that subject from one viewed as being only of academic interest (and, therefore, “very dull and of no practical value whatsoever”) into the presently exciting and active field of “micro-fluidics”.

All of us have gathered here today to celebrate Howard Brenner’s remarkable accomplishments which, over a period of essentially SIX(!) decades, were focused in large measure on what is referred to as “Low Reynolds Number Hydrodynamics,” or “Creeping Flows” or, as Howard used to say in later years with a chuckle, “The Dynamics of Fluids at Rest”! Historically, the reason for using the word “creeping” lies in the definition of the Reynolds number,

\[ \text{Re} = \frac{UL}{\nu} \quad (1) \]

where U is the characteristic speed of the moving fluid, L the characteristic length scale of the object past which the fluid flows and \( \nu \) the kinematic viscosity of the fluid. Since \( (1/\nu) \) is typically very large, \( O(10^{-5}) \) cm\(^2\)/sec for ordinary fluids, and L is, again typically, no smaller than about a cm for objects which one is able to see with the naked eye, small values of Re were historically associated with extremely small speeds U and, thus, outside the domain of anything of much practical importance. In point of fact, I still recall the day, more than 50 years ago, when I attended, for the first time, the meeting of the American Physical Society’s Division of Fluid
Dynamics which, at the time, had a total of exactly 3 Sessions (not simultaneous sessions, mind you, but sequential). The first Session was called “Turbulence I,” the second “Turbulence II” and the third simply “Other”! In fact, in those days, in the opinion of many physicists, the only thing worth remembering about creeping flows was Stokes’ law about the settling speed of a sphere in a very viscous fluid. But, the gradual realization that continuum mechanics is able to model, to an astonishingly degree of accuracy, the dynamics of small bodies down even of molecular dimensions, plus the emergence of the fields of “micro-fluidics” as well as several others with the words “nano” and/or “bio” in their respective titles, have transformed the subject of “creeping flow” from being viewed as, primarily, a mathematical curiosity and limited to “fluids practically at rest”, into a very active field of research full of counterintuitive results of very considerable physical relevance to the world of “small scales,” where the velocities can, actually, be really quite large.

Now, in the early part of his career to which I shall focus in my talk, Howard concerned himself with the mathematical solution of a number of basic steady state hydrodynamic problems under conditions where the Reynolds number Re is not only small but, in fact, exactly zero; so much so, that we used to kid him by telling him that the only value of Re he appeared willing to acknowledge was zero, anything else being irrelevant! Of course, setting Re=0, enormously simplifies the exact Navier-Stokes equations which, as everybody knows all too well, are highly non-linear and fiendishly complicated even at steady state, in that, at Re=0, these reduce to the linear, steady, and reversible Stokes equations which are mathematically analogous to the equations of linear elasticity. But, in spite of such a drastic simplification, these so-called Stokes equations are, aside for special cases, still too cumbersome to solve analytically (or even numerically although to a much lesser extent these days due to the increasing availability of powerful computers and numerical techniques). So, early on in his career, Howard looked around for such special cases involving relatively simple geometries typically involving spheres (sort of toy problems) and then proceeded to generate a large number of new results which, as was realized in later years, turned to be of fundamental as well as practical importance. Examples, to name but a tiny fraction, include his derivation of expressions for:

Figure 1a.

(i) the settling speed of a small sphere in a vertical cylindrical tube with the center of the sphere located at a fractional distance from the longitudinal axis of the cylinder (Fig. 1a); (ii) the force and torque on a sphere moving at constant speed either towards or parallel to a solid wall (Figs. 1b and 1c); and, (iii) the translational
and rotational velocity of a neutrally buoyant sphere near a solid wall in the presence of a simple shear flow (Fig. 1d). It is worth emphasizing that, all these studies could be performed successfully only by somebody having exceptional mathematical skills which, just like the rest of us, Howard had to learn on his own given that, in those early days, the typical chemical engineering curriculum did not include courses requiring the use of any mathematics aside from arithmetic, logarithms and, very occasionally, the evaluation of one-dimensional integrals.

But concurrently, and thanks to his discovery of a remarkable theorem, the so-called “reciprocal theorem,” Howard’s research moved to a higher plane in that he was able to derive results of surprising generality which were often quite unexpected. The theorem itself had appeared in 1897 in a paper in a Dutch journal (written in Dutch) by the great physicist Lorentz, but had been ignored to a large extent by the fluid mechanics community although, curiously enough, it is essentially identical to Betti’s theorem which had been derived 25 years earlier and was both well-known and used widely in solid mechanics (no doubt because, in those days, linear mechanics was already an active and, from the practical point of view, important field, in contrast to fluid mechanics at zero Reynolds number which, as mentioned earlier, was viewed mostly as a curiosity). So, in one of our very last conversations which we had, I asked Howard how on earth he had managed to discover this theorem given that it was never mentioned in any of the fluid mechanics texts typically being used at the time he was a graduate student. Howard then told me that his advisor, John Happel, had collected quite a number of “obscure books,” one of which was by Henri Villat (in French) which Howard began to study in depth and, on seeing a derivation of the reciprocal theorem and being trained as an engineer, he immediately perceived its practical value.

So, let me take a few minutes and explain in a few words what the fuss is all about. Consider a solid body of arbitrary shape, i.e. an “Arbitron” as Howard used to say, fixed in space in the presence of a fluid having a uniform velocity $U=Ue_z$ at infinity where $e_z$ denotes an arbitrary unit vector. Then, on account of the linearity of the Stokes equations, the fact that all the unknown variables are homogeneous functions of degree one in $U$, as well as from dimensional analysis, it is easy to show that $F_z$ the force exerted by the fluid on the “Arbitron,” is of the form:

$$F_z = 6πμLU (R_{ij} e_j)$$  \(2\)

where $R_{ij}$ the dimensionless “resistant tensor,” is a function only of the geometry of the solid body and the boundary conditions on the latter as long as these are linear (note, though, that $R_{ij}$ also depends on how $L$ is chosen in that, for example, it equals, in the case of a sphere, either $δ_i$ or $δ_i/2$ depending on whether $L$ is chosen as the radius or the diameter). This leads immediately to the counterintuitive result (counterintuitive, because our intuition is based on our everyday experience with flows where the Reynolds number $Re$ is far from being small) that, if the flow is
reversed in its direction, the direction of \( F_i \) is exactly reversed with the magnitude of all its components remaining unchanged (see Figs. 2a and 2b). In addition though, in their celebrated volume on *Statistical Physics*, the English edition of which had just appeared while Howard was still a graduate student, Landau & Lifschitz proved that the resistance tensor \( R_{ij} \) is symmetric so that, with reference to figure 3, \( R_{12} = R_{21} \).

Thus, for example, irrespective of the geometry of the *Arbitron*, the “lift” force along the positive 2-axis when the uniform flow is along the positive 1-axis (shown in red in fig. 3a), equals the “lift” force along the positive 1-axis when the flow is along the positive 2-axis; and, similarly, with the other two “lift” forces. This truly amazing result was derived in typical Landau fashion more as follows:

We rewrite eq. (2), which we take as being obvious (which, of course, it is!), by letting \( U = \partial \phi / \partial x_i \), where the “potential” \( \phi \) is linear in \( x_i \). Then we note from Eq. (2), that the “flux” \( F_i \) is a linear function of the “driving force” \( \partial \phi / \partial x_i \), and that, moreover, the scalar product \( F_i \partial \phi / \partial x_i \) equals the rate of mechanical work which is dissipated into heat, i.e. the rate of production of entropy. Thus, eq.(2) satisfies all the conditions for the applicability of Onsager’s theorem of irreversible thermodynamics, according to which \( R_{ij} \) must be symmetric! The truly amazing feature of Landau’s proof is that it does not require any knowledge of the Stokes equations but only of their general properties.

Now, Howard learned about this proof but felt that it would be worthwhile to see if he could prove the symmetry of \( R_{ij} \) starting with the Stokes equations rather than invoking a general theorem from a different, and seemingly unrelated field. This he accomplished using the reciprocal theorem and concluded that \( R_{ij} \) is, indeed, symmetric as shown by Landau in a few words. So, you may ask what is new and, more importantly, what is the big deal? The big deal is that, in addition to “proving” the symmetry of \( R_{ij} \), Howard was able to establish the following truly remarkable result:

Suppose that the stationary *Arbitron* is immersed in the following two flows, denoted by A & B, respectively, and suppose that, for case A, the flow at infinity has the uniform velocity \( U_i \) as was done
previously, while, for case B, the flow at infinity is given by any solution \( u^{\infty}(x) \) of the Stokes equations which has no singularities in the space exterior to the Arbitron. Furthermore, suppose that the Stokes equations have been solved completely for case A so that, \( f_i^{(A)} \), the stress force on the surface of the Arbitron is taken as known for any three mutually perpendicular orientations of \( U_i \). Then, according to Howard’s analysis using the reciprocal theorem:

\[
F_i^{(B)} U_i = \int_S f_i^{(A)}(\vec{x}) u^{\infty}_i(\vec{x}) dS
\]

Consequently, by solving the simpler case A, one is able to obtain, as a bonus, the force \( F_i^{(B)} \), which is usually the only quantity one is interested in, by merely evaluating a surface integral for any undisturbed Stokes flow \( u^{\infty}(x) \), rather than solving from scratch a new and immensely more complicated boundary value problem. Furthermore, as Howard showed, one can construct, via the reciprocal theorem, a host of other symmetry relations in addition to those of \( R_{ij} \).

For example, when the flow at infinity, rather than being uniform, is a pure strain of the form \( G_{ij}x_j \), with \( G_{ij} \) being symmetric and having units of inverse time, we have that, in lieu of (2),

\[
F_i = \mu L^2(T_{ijk}G_{kj})
\]

where the 3rd order tensor \( T_{ijk} \) has several symmetry properties analogous to those of \( R_{ij} \). All these and many other such general results, e.g. the extension of Faxen’s “law” to non-spherical particles as well as to a suspension of hydrodynamically interacting spheres, were put to use several years later by John Brady and Georges Bosis when developing their well-known Stokesian Dynamics numerical technique for solving complicated problems involving particle motions in Stokes flows.

About this time, however, Howard also started investigating several basic fluid mechanical problems under conditions of small but non-zero values of Reynolds number, often in close collaboration with Raymond Cox, an exceptionally gifted applied mathematician who had just received his Ph.D. at Cambridge. Using, in large measure, the method of inner and outer expansions, which had been developed earlier, first at Caltech and then at Cambridge, the pair generated a number of remarkable results of which the one given below is but one example.

Consider once again the Arbitron, fixed in space in the presence of the uniform flow at infinity, \( U_i = U\delta_{ij} \), and with prescribed boundary conditions which are linear and independent of the Reynolds number \( Re \). In that case, when \( Re = 0 \) and in view of (2), \( F_i \), the component of the Stokes force in the direction of the uniform flow equals \( 6\pi\mu LU_{ij}R_{1i} \). Brenner & Cox then showed that, at small but non-zero values of \( Re \)

\[
F_i = 6\pi\mu L[U_{ij} + 3(Re)/16][3R_{ij}R_{-ij} - (R_{ij})^2] + (9/40)(Re)^2 \log(Re)[R_{ij}R_{-ij} + \text{O}(Re^2)]
\]

and similarly when \( U_i = U\delta_{ij} \) or \( U\delta_{ij} \), where the \( R_{ij} \)'s are the components of the Stokes resistance tensor introduced earlier. Thus, knowledge of the Stokes solution allows one to obtain, for any orientation of the uniform flow \( U_{ij} \) the leading order corrections to the drag, \( F_e \), due to inertial effects without any additional calculations. It is worth remarking that, to this order, the drag on the Arbitron remains unaffected if the flow is reversed.

Analogous results were derived by Brenner & Cox for the corresponding “lift” forces except that, in general, the \( \text{O}(Re) \) correction to the Stokes lift contains an additional term, not related to the elements of \( R_{ij} \) which, as shown by Gary Leal some years later, can also be obtained more directly via the reciprocal theorem.
My last example is, perhaps, the most striking. Consider a heated \textit{Arbitron} in the presence of a uniform flow at infinity $U$ from, say, from left to right. The temperature $T$ of the solid body is held fixed at unity while that of the fluid at infinity is taken to be zero. The physical properties of the fluid are taken to be independent of temperature, and the Reynolds number is assumed small enough so that the fluid velocity satisfies the Stokes equations. At steady state then, the energy equation reduces then to:

\[
\frac{\partial^2 T}{\partial x_i^2} = \text{Pu} \frac{\partial T}{\partial x_i} \tag{5}
\]

where $P$ is the Peclet number which, in principle, could be made arbitrarily large. Also, we suppose that $u_i(x)$ is known from the solution of the Stokes equations. We wish to determine to what extent the overall heat transfer rate, as given by the Nusselt number, is altered if the sign of $U$ is reversed so that the flow at infinity is from right to left. At first glance, the problem appears to be exceedingly messy even though we know that $u_i$ is reversed everywhere, because of the existence of thin thermal layers, especially at very large $P$, which are located in the forward section of the \textit{Arbitron} if the flow is from left to right and, at its rear end if the flow is reversed. In fact, one might expect that the answer would depend on $P$ as well as on the geometry of the \textit{Arbitron} and on the details of the fluid velocity profile. In point of fact, as shown by Howard via the reciprocal theorem, the two Nusselt numbers are \textbf{identical} for any value of $P$! The same result can also be shown to apply for any undisturbed flow $u_i^{\text{oe}}(x)$ satisfying the Stokes equations and having no singularities within the domain external to the \textit{Arbitron}, as well as in the analogous case of mass transfer to the bulk from a reacting surface involving a single reactant, as long as the surface reaction rate is first order in the concentration of the reactant.

Several of Howard’s early results found their way in the book \textit{Low Reynolds Number Hydrodynamics} by Happel and Brenner which first appeared 49 years ago and whose 50th birthday all of us looked forward to celebrating next year, especially so Howard. It is of interest to point out that the book was not exactly a best seller when it first appeared on the scene. It was much too “heavy” mathematically for most engineers, the field itself was considered far too narrow to be much more than a curiosity and the review which it received in the JFM was anything but positive the book having been judged as being “boring.” Yet, it is now acknowledged as the bible in the field for generations of rapidly and steadily increasing numbers or researchers and, although it has not been revised in any meaningful way, its annual sales have been on a positive slope ever since its inception, which is something that cannot be said for the large majority of 50-year old books.

On a personal tone, Howard and I had been close friends for almost 60 years. He was always there to help me when I needed him and vice versa. I feel privileged to have known him and I shall forever treasure the memory of our friendship and of the wonderful times we had together talking science and, later on, marveling at the numerous exciting developments in a vibrant field once deemed to be so “dull”!
Howard Brenner and Blue Pumps

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I bought my first copy of *Low Reynolds Number Hydrodynamics* (LRNH) in 1970; within 10 years the binding was held together with tape, and now I am afraid to move it for fear that the pages will fall out (Figure 1). The wear and tear on this classic reference says more than words about the respect I have for Howard Brenner and his work. He changed my life and that of others; he inspired us to reach higher.

I first met Howard in 1969 at the University of Illinois, where I was a Ph.D. student. I was trying to measure the hindered diffusion of nanoparticles through well-defined, cylindrical pores in membranes. The goal was to test hydrodynamic theories for the relationship among pore size, particle size, and diffusion rate. I was thrilled to have the opportunity to actually meet the author of *LRNH*. I was inspired by his seminar; it was then that Howard became one of my academic heroes and remains so to this day.

In the early 1970s John Quinn and I developed a theory for “hindered diffusion” of spherical particles in cylindrical pores. We used numerical calculations published by Howard and his students for the hydrodynamic effect of the pore wall, allowing for off-centerline positions of the particle. We did a numerical matched-expansion of their calculations for motion near the centerline and near a flat wall. While our calculations were correct, the results were in tabular form. Howard and his doctoral student Lawrence Gaydos re-solved the problem analytically using asymptotic expansion techniques and obtained a short, simple algebraic equation for the hindrance of the pore, which matched our numerical results perfectly. Today the Brenner-Gaydos expression is cited, not the Anderson-Quinn numerical results—lesson learned.

Howard and I were colleagues together at Carnegie Mellon University (CMU) for one year when I joined the faculty in 1976. He, Dennis Prieve, and I formed a great friendship that year; Dennis and I looked to Howard as our leader and mentor. We instituted a weekly seminar series in the evening preceded by dinner and several glasses of wine; we remember the dinner more than the seminar because Howard liked to talk about his philosophy of scientific research, becoming more emphatic with each glass of wine. It was great fun.
At one dinner Howard mentioned his new interest in doing experiments (actually, having a student do the experiments). The student was to measure the thermodynamic partitioning of amphipathic particles between water and a non-polar solvent. When he calculated the energy difference of one particle between the two phases, he obtained 150kT. Howard’s approach would have the student measure the number of particles in each phase at equilibrium. Dennis and I pointed out that exp(-150) is a very small number, and the graduate student would have to spend a very long time to find one particle in the non-favorable solvent. After arguing with us for some time, Howard was finally swayed and dropped the idea. He never suggested an experiment after that.

Howard’s sense of humor is legendary. I audited his course on fluid mechanics. At one point in a lecture, one student seemed to be frustrated by Howard’s focus on polyadic notation and the Stokes equations. He asked, “Professor Brenner, all this math is great, but what does a pump look like?” Howard calmly reflected on the question and then responded with a straight face: “Well, some are red, some are green, and even others are blue.” Muffled laughter, no more questions.

Actually I am surprised that Howard knew the color of a pump. Figure 2 shows him as a young faculty member in the unit operations lab at New York University. Close inspection reveals some confusion in his eyes. He told me the last experiment he did was as an undergraduate student in a qualitative analysis course in chemistry. The unknown element he was trying to identify was cadmium, but after many steps he ended up with a solid in the bottom of the test tube that, upon a melting test and visual inspection, was identified as sodium chloride. End of experiments for Howard Brenner.

Howard was known for his insistence on using polyadic notation and being generally dismissive of Cartesian tensor notation. But his disdain for tensor notation was not absolute. One day I walked into his office and asked him to prove a differential operation on a particular dyadic. He looked at the identity for a while, then with pencil and paper scribbled some notes, shielding them from my view. When I insisted on seeing his notes, he reluctantly showed me the proof in tensor notation. We both laughed.

The Chemical Engineering Department at CMU is internationally recognized for its programs in colloids, polymers, and surfaces (CPS). Howard was one of the founders of the program and promoted it to the senior administration at CMU in the early 1970s. His vision was extraordinary; not only did CPS have a great impact on CMU, it has also become one of the mainstream disciplines of chemical engineering.

Those who inspired and encouraged us are never gone; they are in our minds and our hearts forever. Howard Brenner showed me how to combine rigorous analysis with conceptual thinking, how to enjoy research as a hobby as much as a job, how to argue and maintain friendships, and how to laugh in the process. Thank you, Howard.

Figure 2. Professor Brenner inspecting laboratory equipment at New York University (ca. 1964)

Figure 3. Experimentalist (John Quinn) and theoretician (Howard Brenner) sharing stories (2005)
Macrotransport Theory and the Fick-Jacobs Approximation

The cancer of heuristic arguments and their treatment by macrotransport therapy

Kevin D. Dorfman and Ehud Yariv

Department of Chemical Engineering and Materials Science
University of Minnesota
and
Faculty of Mathematics, Technion
Transport in slowly varying channels

\[ Y_+ = \epsilon L h_+(X/L) \]

\[ Y_- = \epsilon L h_-(X/L) \]

Fick-Jacobs approximation for the transport projected to 1-D

\[ \frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left[ D_0 A(x) \frac{\partial}{\partial x} \frac{C(x, t)}{A(x)} \right] = \frac{\partial}{\partial x} \left[ D_0 \left( \frac{\partial C}{\partial x} - \frac{C \, dA}{A \, dx} \right) \right] \]

Transport in slowly varying channels

\[ Y_+ = \epsilon L h_+ \left( \frac{X}{L} \right) \]

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"Entropic" force

Who did it first? (compliments of L. Dagdug)

Who did it first? (compliments of L. Dagdug)

A. Fick, Poggendorff’s Annalen. (1855) 94, 59-86  
This is a picture of the student office circa 2000. Howard let us clean it up but did not want to know about what we did. One time he came in and asked us why we were getting rid of a book. We said it was in Russian. He said that was no problem, since we could read the equations!
This problem came about at a workshop in Germany shortly after Howard’s death. Another interesting thing that happened after Howard died was the submission of another paper from beyond the pale.
The real transport question

Fick-Jacobs approximation for the transport projected to 1-D

\[ \frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left[ D_0 A(x) \frac{\partial}{\partial x} \frac{C(x, t)}{A(x)} \right] = \frac{\partial}{\partial x} \left[ D_0 \left( \frac{\partial C}{\partial x} - \frac{C}{A} \frac{dA}{dx} \right) \right] \]

Correction for a finite wavelength perturbation to the channel width

\[ \frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left[ D(x) A(x) \frac{\partial}{\partial x} \frac{C(x, t)}{A(x)} \right], \]

What is the correct form for the axial diffusion coefficient?

Zwanzig, J. Phys. Chem. 96, 3926 (1992)
Proposals for the diffusion coefficient

Heuristic model (Zwanzig)

\[ D(x) = \frac{D_0}{(1 + \epsilon w')^2} \]

Irreversible thermodynamics (Reguera & Rubi)

\[ D(x) = \frac{D_0}{(1 + \epsilon w')^{1/3}} \]

Systematic perturbation theory to first order in \( w' \) (Kalinay & Percus)

\[ D(x) = D_0 \left( \frac{\arctan(\epsilon^{1/2} w')}{\epsilon^{1/2} w'} \right) \]

Zwanzig, J. Phys. Chem. 96, 3926 (1992)
Reguera and Rubi, Phys. Rev. E 64, 061106 (2001)
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Zwanzig, J. Phys. Chem. 96, 3926 (1992)
Reguera and Rubi, Phys. Rev. E 64, 061106 (2001)
Macrotransport approach

Lubrication-type solution of the macrotransport equations

Regular perturbation expansion for the effective diffusion coefficient

\[ d^* = d_0^* + \epsilon d_1^* + \epsilon^2 d_2^* + \ldots \]

Leading order term is the result for \( D(x) = \text{constant} \)

\[ d_0^* = \frac{1}{\langle 1/w \rangle} \]

First correction agrees with asymptotic expansions for all \( D(x) \) proposals

\[ d_1^* = -\frac{1}{3} \frac{\langle w'^2/w \rangle}{\langle 1/w \rangle^2} \]

Second correction finds the correct \( D(x) \) proposal

\[ d_2^* = \frac{4}{45} \frac{\langle w'^4/w \rangle}{\langle 1/w \rangle^2} + \frac{\langle w'^2/w \rangle^2}{9 \langle 1/w \rangle^3} + \frac{\langle w w''^2 \rangle}{45 \langle 1/w \rangle^2} \]

Lubrication-type solution of the macrotransport equations

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\]

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Piled Higher and Deeper

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Piled Higher and Deeper by Jorge Cham

What you think of your Professor vs. Time

- He or she is a genius!!
- OK, they're one of the best, but maybe not the best.
- They're "OK." I could do better.
- Do they even know what they're doing??
- At least they came to my thesis defense.
- Nice guy. (or gal)

1st YEAR  2nd YEAR  3rd YEAR  4th YEAR  5th YEAR  TIME

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WHAT YOU THINK OF YOUR PROFESSOR vs. TIME

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1st YEAR 2nd YEAR 3rd YEAR 4th YEAR 5th YEAR

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Eating the Bottle
David Edwards
Harvard University
Instructions for using the Atlas Slide Rule.

Printed in U.S.A.

Gibson Slide Rule Co., Box 1237, St. Louis, Mo.
(Slide Rule Made Since 1818.)
Meet the Faculty

Professor Brenner

While many people today are under the impression that engineers are little more than machines which pop out answers using a handbook formula without knowing the theory behind the work, it is refreshing and reassuring to find brilliant young men like Dr. Howard Brenner of the Chemical Engineering Department who, while realizing the importance of specific formulas, feel that the future rests in their ability to be free thinkers. Presently an Assistant Professor, Dr. Brenner has been teaching at New York University for five years. In the undergraduate division he teaches a course in Unit Operations (Ch. E. 50) and a course in Thermodynamics (Ch. E. 77). He teaches an advanced thermodynamics course in the graduate division as well as a course in Fluid-Solids Dynamics and another in the Theory of Transport Processes. He enjoys teaching and appreciates the freedom teaching allows. In instructing young is allowed complete freedom. The company expects that although no immediate applications may be apparent for many ideas which he submits, in the long run these ideas will become useful and profitable for the company.

Dr. Brenner does a lot of reading and buys many books. He advises his students to do the same. The subjects of these books are not necessarily directly related to his work but often concern many other subjects. He enjoys material concerning psychology and logic in addition to subjects which are quantita-
EATING THE BOTTLE: DRAWING A HOWARD BRENNER LESSON—DAVID EDWARDS

The Use of Instrumental Analysis

By Charles J. Harell, Cyril A. Tennyson, Howard Brener, and Frank E. Henry

Reprinted from SOAP AND SANITARY CHEMICALS, Nov., Dec., 1928
Thanks Howard

650 East Kendall St
Cambridge MA
lelaboratoirecambridge.com
cafefeartscience.com
Left to right: Howard Brenner relaxing at home with proverbial pipe; lecturing in plaid; receiving an honorary degree from Clarkson University
Reflections on Brenner’s Bi-velocity Fluid Mechanics

Joe Goddard

Department of Mechanical & Aerospace Engineering
University of California, San Diego
La Jolla, California, USA
As one of his numerous important contributions to physicochemical hydrodynamics, Howard published some two dozen papers in the eight-year period 2004-12 on this topic and considered it one of his legacies.

The central idea is that (barycentric) mass-centered velocity defining fluid inertia is not generally the same as the (“volume” or “work”) velocity, whose gradient governs viscous stress.

The distinction becomes important for inhomogeneous fluids, where the second velocity not only governs stress but also influences heat flux.

Incorporating the latter into the classical Navier-Stokes/Fourier equations leads to effects such as Maxwell thermal stresses, already known in rarefied gas dynamics.
Brenner’s Papers Related to Bi-velocity Fluid Mechanics


Papers Related to Bi-velocity Fluid Mechanics (cont’d)


Relation to Contemporary Continuum Mechanics

• Brenner’s work points up a logical flaw in the modern literature on continuum mechanics, with volume-based mappings of continuous sets of “material points,” sometimes regarded as barycentric but at other times regarded as “dyna-centric,” i.e., centers of force.

• The latter interpretation is essential to the fundamental principle of virtual work, by which forces are actually defined in terms of kinematics.

• The Speaker\(^1\) shows that the ambiguity reflects the emergence of non-locality & breakdown of the Coleman-Noll “simple material” model for finite Knudsen numbers \(||\lambda \nabla||\), i.e., large gradients \(\nabla\) on material length-scale \(\lambda\).

• A general linear theory leads to additional terms, with, e.g., implications for thermo-acoustic wave propagation, effects which are being investigated as an extension of the work of Davis and Brenner, 2012.

Conclusion and Outlook

• Brenners’s bi-velocity fluid mechanics provides one plausible representation of higher-gradient non-equilibrium effects in inhomogeneous fluids.

• Other terms may arise, and it seems worthwhile to continue Howard’s investigation of such effects in various physical problems, e.g., in gas dynamics, liquids confined to small geometries (microhydrodynamics), and thermo-acoustic wave propagation.

• Howard Brenner’s vigorous and lifelong focus on fundamentals once more bears fruit, raising basic questions about fluid mechanics and the more general mechanics of continua.
In Honor of Howard Brenner

Arthur J. Goldman

November 16, 2014

It is my privilege to be here today to honor Howard by telling you of my personal experiences as a young man who had the good fortune to become one of the first of Howard’s protégés. Thanks to John Anderson for making my appearance here possible and for a serendipitous conversation he and I had that reconnected me with Howard very late in his career; but more on that later. Often we get to express our feelings about those important people in our lives after they pass, like we are doing today. I consider myself fortunate that I was able to share those feelings directly with Howard while he could appreciate it and that I could hear directly from him how important our relationship was to him as well.

My story begins in 1957 as a recent Chem. E. graduate of CCNY, taking graduate courses at night at NYU while employed full-time during the day. I was looking to strengthen my theoretical science foundation and develop math skills. Fortunately I found Howard! Over two years I enrolled in the four courses he taught in thermodynamics and theory of transport processes. He not only deeply understood the subject matter but could teach it so that I understood it, appreciated it, and could apply it. Howard was an impressive presence in front of a classroom. He radiated confidence and lectured without notes. He was patient; he would answer every question and elaborate in ways that gave new insight. I was hooked on his approach, his style, and his depth of knowledge. This was my kind of chemical engineer, a role model.

In 1962 I passed my doctoral qualifying exam and began thinking about how I would satisfy the research requirements for the doctoral degree. Howard agreed to be my advisor. He assured me that he would define for me a problem worthy of investigation, but we would wait until that time came to get to specifics. Unbeknownst to me Howard was in the midst of writing his first book with John Happel, *Low Reynolds Number Hydrodynamics with Special Applications to Particulate Media*, published in 1965. The die was being cast.
IN HONOR OF HOWARD BRENNER—ARTHUR J. GOLDMAN

I began my full-time research at the University in 1964 as an AEC Fellow. Howard had defined for me a problem to work on—to come up with a general theory to characterize the steady settling motion of arbitrary-shaped particles at low Reynolds Number. I completed the project in about 6 months, but that wasn’t sufficient time to satisfy the University residency requirement so Howard defined another problem, actually a series of problems involving low Reynolds Number flow of one or two spherical particles moving in either a quiescent fluid or in a fluid shear field in proximity of a wall. There were already published a variety of exact solutions for special cases, approximate solutions, and experimental results, but there was a need for exact general solutions to provide a basis for comparison and for developing exact solutions for other particle flow conditions. I managed to solve the problems, wrote them up, defended the thesis, received my degree, and moved on. At that time, unbeknownst to me, Howard was getting ready to move on himself to Carnegie Mellon but there was still unfinished business—publication of the results of my research in a professional journal. I drafted three articles for publication and Howard, in his inimitable and recognizable style, edited and submitted them to Chemical Engineering Science. The rest is history. Together those three papers by Goldman, Cox, and Brenner have accrued over 1,700 citations to date and counting, and have become foundational for researchers in the fields of biophysics and nanotechnology, something I was unaware of until recently. But more on that later.

Let’s fast-forward 44 years to the fall of 2010 at a meeting of the Chicago Council On Science and Technology, of which John Anderson and I are board members. During the course of the meeting John mentioned that at an earlier time he was Dean of the School of Engineering at Carnegie Mellon. That reminded me of Howard; perhaps they knew each other. So after the meeting I engaged John and asked if he knew Howard Brenner from that connection. He was very pleased and impressed by the coincidence and proceeded to tell me about their close friendship that continued even after each had moved on. John asked me about my background and I told him that Howard was my thesis adviser. At that point John’s face lit up. He realized something that was of great significance to him, about which I had no idea until he explained it. The Goldman, Cox, Brenner papers that were based on my thesis work were, by John’s own admission, a foundation for his research. He was excited to meet me, “The Goldman” of particle dynamics notoriety. When we finally said goodbye he told me he would write to Howard and tell him of this exciting coincidence. I was greatly surprised by this unsolicited, unexpected feedback about something I did a long time ago that had remained buried in my memories, and which I thought was meaningful only to me. I came to realize it was more meaningful and significant than I ever realized.

John did write to Howard, Howard wrote to me, and we enjoyed further exchanges of correspondence. I want to read a few excerpts from those exchanges that capture Howard’s and my own personal words and feelings at the time.

Hi Art:

How nice to hear about you via John Anderson. Though out of touch with you for these many years, you’ve often come to mind. Goldman, Cox, and Brenner. What a combination! Many people have approached me for a copy of your thesis, since a lot of the details of your calculations were left out of our published papers.

Howard
Dear Howard,

I can’t tell you how much this renewed contact means to me and has impacted me emotionally. As you say, we get increasingly nostalgic as we age, and I am an easy target. I am 5 years younger than you, but we both are in that category where reminiscing and renewing contacts have taken on enormous significance.

Ours is a mutual admiration society. I can say unequivocally that you were the most important person in my entire academic experience in helping to shape my career, both in the classroom and in my research. I gained more insight and learned more from your thermodynamics classes and your heat, mass, momentum classes than any that I can remember. You were a superb teacher; your book with Happel is, indeed, foundational. And your recommendation for me to replace you as an adjunct to teach heat, mass, momentum transfer was for me a learning and growth experience that was unique and much appreciated.

Art

Dear Art:

I was as deeply moved by your warm letter as you were by mine. Indeed, I read your kind letter to my wife. I remember you well, and was very pleased to have the opportunity to renew acquaintances, especially at this stage of the game where reviewing past relationships acquires heightened interest.

Long-interrupted contacts with those who played significant roles during formative times in one’s former life are highly prized. Student-mentor interactions are symmetrical with respect to the impact of these contacts on both parties. I always regarded you as the highlight of my NYU career as a mentor. Recollections of rewarding experiences in one’s early years invariably come to the fore in the twilight of one’s life, when nostalgia pervades one’s thoughts. In a career that spanned some 55 years (beginning in 1955 as an Instructor at NYU) I always numbered my interactions with you as being among my most rewarding experiences as an advisor.

Fortunately, Alzheimer’s and other major health issues have not yet intruded into my life in any major way. This enables me to continue at a pace that is currently satisfying to me. Though retired, I’m able to work without benefit of student collaborators since a significant portion of my research activities was always conducted on my own, independently of the need for (and pleasure associated with) such collaborations. I do, however, miss the computational and graphical skills possessed by young students, having, myself, never developed any capabilities in these areas. (Assuming that more youthful persons like yourself possess some of these technological skills in abundance, I await your volunteering to pick up where you left off at NYU, enabling us to resume our fruitful collaboration.) Currently, I am intensely involved—as I have been for much of the past 8 years—in attempting to convince skeptical colleagues and unpleasant referees that volume needs to be added to the existing list of physical properties capable of being transported through fluids (namely heat, mass, momentum, and species).

Incidentally, the first edition of (the Low Reynolds Number Hydrodynamics) book is still in print after 45 years and can be purchased as a paperback from Elsevier, albeit for some outrageous amount of money. The source of the book’s longevity is not significantly different from that underlying that of the GCB papers—primarily its current use in biophysical applications, where low Reynolds number flows dominate.

In summary, you can be very proud of your professional impact on the field of hydrodynamics, and on the lives of those who have found your youthful investigations to be both useful and interesting. Only rarely does Ph.D. research achieve this citation status or longevity. Indeed, in a 50+ year academic career I personally know of no other chemical engineering publication in its field to have done so.

I will look forward to catching up with you in Chicago, or elsewhere, in the near future.

Howard
As a post-script, Howard and I agreed to continue our communications by phone. He promised that he would contact me when he visited Chicago, which he did several times a year. Later that winter he did make the trip and we arranged to meet for dinner. Once we engaged in conversation his voice and classic New York accent took me back to those good old days. He shared with me personal reflections on his career that revealed inner feelings not previously evident. He told me how he always felt his role as mentor was a serious, lifelong responsibility that continued beyond that of teacher-student, a fact that I’m confident his disciples will attest to in the remarks that follow. He told me that he was rather selective in whom he chose to work with as a research advisor and mentor, speculating that it numbered only around 30 over his career, which surprised me but also made me proud to be a member of that exclusive club. I guess it reflects that he basically marched to his own drummer and lived his life his way. Although not one to blow his own horn, I am confident that in his later years he was able to enjoy the satisfaction and gratification of a meaningful and purposeful career that was superbly done and will be long remembered.
Howard Brenner’s contributions at MIT

Klavs F. Jensen

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Cambridge, MA  02139
Howard Brenner, Willard Henry Dow Professor Emeritus of Chemical Engineering, passed away on Monday, Feb. 17. He was 84.

Howard’s extraordinary and accomplished academic career spanned over 60 years. Considered one of the world’s foremost theoreticians in the transport properties of flowing suspensions and multiphase systems, he had a profound impact on the profession through his broad and fundamental research on low Reynolds number fluid-particle hydrodynamics, microfluidics, complex fluids, interfacial transport processes and emulsion rheology, multiphase flow and transport processes in porous media, generalized Taylor dispersion phenomena, and macro transport processes.

Howard was co-author of three textbooks, including Low Reynolds Number Hydrodynamics, a classic textbook and one of the most widely cited books in fluid mechanics worldwide. He also covered interfacial phenomena (Interfacial Transport Processes and Rheology with David A. Edwards and Darsh T. Wasan 1991), and multiple length- and time-scale homogenization schemes (Macrotransport Processes with David A. Edwards 1993). Howard directed the research of a large number of Ph.D. students, master’s students, and postdoctoral fellows in the general areas of fluid mechanics and transport processes. A number of these former students now hold or have held academic appointments on a variety of faculties, including: chemical engineering, mechanical engineering, aerospace engineering, civil engineering, engineering mechanics, and physical chemistry. During his career he singly and jointly published over 200 technical papers, 35 chapters in books, monographs, and proceedings volumes, and presented more than 500 invited seminars and professional lectures.

Raised and educated in New York City, Howard graduated from Brooklyn Technical High School in 1946, where he took its “chemical course.” Prior to attending college he worked for almost a year for a chemical consulting firm in lower Manhattan. Howard received his undergraduate degree from Pratt Institute (1950), and both his Master’s (1954) and Sc.D. (1957) from New York University, both in chemical engineering. He went on to serve on the chemical engineering faculties of NYU (1955-1966), Carnegie Mellon (1966-1977), University of Rochester (1977-1981) where he was department chair, and finally at MIT (1981-2005), where he was the Willard Henry Dow Professor of Chemical Engineering.
Howard was an enthusiastic researcher whose lifetime work never stopped – certainly not in retirement. Until 3 days before his death, despite tremendous physical challenges, he was making the final revisions on a paper that reflected the culmination of almost ten years' work, much of it done after he became emeritus. Of this project, he said that he could always continue to find i's to dot or t's to cross, but that, in essence, he was done with what he believed to be a seminal piece that overturned a theoretical underpinning of fluid dynamics several hundred years old.

In the special issue of Chemical Engineering Communications honoring Howard's 80th birthday in 2009, his former students recounted, “HB held everyone to high standards, especially himself, and he was never shy in getting into very deep discussions of the most recent or subtle concepts he was working on (usually, the two went hand in hand). Such discussions were not a rare event; indeed, we can distinctly remember the knock on the door each afternoon that announced his arrival to the students' office. With a smile on his face, HB would enter the room, take a chair at the center, and launch into an in-depth discussion of the latest theories under development. These frequent and informal scientific discussions were the epitome of academic endeavor and left an indelible impression on everyone who had the opportunity to work with him.” Howard took great pleasure in encouraging his students to pursue the pure sciences in their career paths, believing that to be engaged in a world of ideas was a gift they might be as lucky as he to experience.

Howard was recognized by numerous honors and awards, including election to the National Academy of Sciences, the National Academy of Engineering, the American Academy of Mechanics, the American Academy of Arts and Sciences, and the American Association for the Advancement of Science. As well as being an elected fellow, he received all three major society awards from the American Institute of Chemical Engineers, each representing a different category of accomplishment: the Alpha Chi Sigma Award (1976) for "outstanding accomplishments in fundamental chemical engineering research," the William H. Walker Award (1985) for "outstanding contributions to the chemical engineering literature", and the Warren K. Lewis Award (1999) for "distinguished and continuing contributions to chemical engineering education." Other awards include the 2001 Fluid Dynamics Prize from the Division of Fluid Dynamics of the American Physical Society, for “his
outstanding and sustained research in physic-chemical hydrodynamics, the quality of his monographs and textbooks, and his long-standing service to the fluid mechanics community,” the 1995 General Electric Senior Research Gold Medal Award of the American Society for Engineering Education, the 1980 Bingham Medal of the Society of Rheology, and the 1988 American Chemical Society Award in Colloid or Surface Chemistry as well as their 11th Annual Honor Scroll in 1961.

During a symposium to honor his 70th birthday in 1999, Howard observed, “coming from a family of more short-lived ancestors [his mother died in her late 60s and his father in his early 70s], I will settle for seeing my colleagues at an 80th birthday celebration, which he did.” Fortunately for the world of science and his family, he lived to be productive for many years beyond his 70th birthday, passing away just a month shy of his 85th birthday on March 16, 2014.

*Howard inducted into the National Academy of Sciences*
Howard Brenner at MIT

- Joined MIT July 1, 1981 as Willard Henry Dow Professor
- Supervised 19 Ph.D. students, 9 of whom went on to academic careers
- Had 17 postdocs along with a number of postdoc and visiting students
- Contributed ~190 papers and 2 books
  - *Macrotransport Processes* with David A. Edwards, 1993
- ~400 invited seminars and professional lectures
Howard Brenner at MIT

• Received numerous awards, including:
  – 1980 Bingham Medal of the Society of Rheology
  – National Academy of Engineering
  – 1985 William H. Walker Award
  – 1988 American Chemical Society Award in Colloid or Surface Chemistry
  – American Academy of Arts and Sciences
  – 1995 General Electric Senior Research Gold Medal Award of the American Society for Engineering Education
  – 1999 Warren K. Lewis Award
  – 2000 National Academy of Science
  – 2001 Fluid Dynamics Prize from the Division of Fluid Dynamics of the American Physical Society
  – 2004 Honorary DSc from Clarkson University

• Retired October 31, 2005 after 50 years in academia

• Remained active until his death early this year
Research activities during the period under discussion involved several major foci. The most prominent of these includes research in the following areas:

- **Macrotransport processes**, involving flow, dispersion, and chemical reactions in geometrically complex systems, including porous media, for example.
- **Physicochemical hydrodynamics** of fluid-particle systems.
- **Interfacial transport processes** and rheology, especially involving surfactants and other surface-active agents.

The productivity and size of the research group under the writer's direction is reflected in the significant number (and, hopefully, quality) of publications resulting from the activities of this group during the past 8 1/2 years. These data are detailed in Appendix A, which furnishes a complete list of the 91 research and related professional publications which have been published, accepted for publication, or submitted for publication during the period of time covered by this report.
Howard Brenner as Mentor and Colleague
Howard Brenner: “Coming from a family of more short-lived ancestors, I will settle for seeing my colleagues at an 80th birthday celebration.”
Presenting his new work to his colleagues on his 80th birthday

Howard presenting his work on the shortcomings of the Navier-Stokes equations to his Chemical Engineering colleagues on his 80th birthday.

Howard proposed a new approach to fluid mechanics based on the concept of two different velocities: the mass-based (Eulerian) mass velocity derived from the classical notion of mass transport, and the fluid-based (Lagrangian) volume velocity associated to the motion of individual particles (molecules).
Conduction-only transport phenomena in compressible bivelocity fluids: Diffuse interfaces and Korteweg stresses

Howard Brenner
Department of Chemical Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139-4307, USA
(Received 10 March 2014; published 29 April 2014)

“Diffuse interface” theories for single-component fluids—dating back to van der Waals, Korteweg, Cahn-Hilliard, and many others—are currently based upon an ad hoc combination of thermodynamic principles (built largely upon Helmholtz’s free-energy potential) and so-called “nonclassical” continuum-thermomechanical principles (built largely upon Newtonian mechanics), with the latter originating with the pioneering work of Dunn and Serrin [Arch. Ration. Mech. Anal. 88, 95 (1985)]. By introducing into the equation governing the transport of energy the notion of an interstitial work-flux contribution, above and beyond the usual Fourier heat-flux contribution, namely, $j_q = -k \nabla T$, to the energy flux, Dunn and Serrin provided a rational continuum-thermomechanical basis for the presence of Korteweg stresses in the equation governing the transport of linear momentum in compressible fluids. Nevertheless, by their failing to recognize the existence and fundamental need for an independent volume transport equation [Brenner, Physica A 349, 11 (2005)]—especially for the roles played therein by the diffuse volume flux $j_v$ and the rate of production of volume $\pi_v$ at a point of the fluid continuum—we argue that diffuse interface theories for fluids stand today as being both ad hoc and incomplete owing to their failure to recognize the need for an independent volume transport equation for the case of compressible fluids. In contrast, we point out that bivelocity hydrodynamics, as it already exists [Brenner, Phys. Rev. E 86, 016307 (2012)], provides a rational, non-ad hoc, and comprehensive theory of diffuse interfaces, not only for single-component fluids, but also for certain classes of crystalline solids [Danielski and Wierzbka, J. Phase Equilibr. Diffus. 26, 573 (2005)]. Furthermore, we provide not only what we believe to be the correct constitutive equation for the Korteweg stress in the class of fluids that are constitutively Newtonian in their rheological response to imposed stresses but, equally importantly, we establish the explicit functional forms of Korteweg’s phenomenological thermocapillary coefficients appearing therein.

DOI: 10.1103/PhysRevE.89.043020   PACS number(s): 47.10.ad
Left to right:

Howard’s early paper with John Happel; one of Howard’s first papers “Silver Dips” (with wife, Lorraine, as model); John Quinn’s 70th Birthday Tribute
Multiple Reflections* on the Life and Achievement of a Great Teacher and Scholar

Sangtae Kim
Distinguished Professor
Purdue University
School of Chemical Engineering

*Should be “Multipole Reflections” but spelling checker at work
Closing Remarks: After All Has Been Said

The great roles of Howard Brenner (and how they helped me)

- Teacher
- Scholar
- Mentor
- Editor (Butterworth)
- Role Model
- ...and... (insert surprise role, 1995)

Landlord (Berkeley sublet)
My First “Meeting” with Howard Brenner
(small library, Spalding Laboratory)


The Stokes resistance of an arbitrary particle—IV
Arbitrary fields of flow
H. Brenner†
Department of Chemical Engineering, New York University, Bronx 53, New York

... via the Lorentz Reciprocal Theorem ...

These integrals may be evaluated by Horson’s theorem. Thus, setting \( \psi = u \) in (7.12) and substituting the result into (7.22), one obtains for the force on a stationary ellipsoid

\[
F' = \mu K \left[ u_o + \frac{1}{3!} (D^2 u)_o + \frac{1}{5!} (D^4 u)_o + \frac{1}{7!} (D^6 u)_o + \cdots \right]
\]  (7.24)
Generalization of Faxen’s Laws

Singularity solution with $U$ as translational velocity (in index notation)

\[ v_i = \mu K_{jk} U_k \mathcal{F}\{\mathcal{G}_{ij} / (8\pi\mu)\} \]

Same functional $\mathcal{F}$ in the Faxen’s Laws

\[ F_i = \mu K_{ij} \mathcal{F}\{u_j\} - \mu K_{ij} U_j \]

The neutrally-buoyant case: Faxen’s Law for translational velocity (Gibb’s notation!)

\[ U = \mathcal{F}\{u\} \]

Method of Reflections: inter-locking slender tori
Howard’s influence and legacy continues (July 2015)
Ellipsoid revisited (see S. Kim, I&EC Research, 2015)

The Stokes resistance of an arbitrary particle—IV
Arbitrary fields of flow
H. Brenner†
Department of Chemical Engineering, New York University, Bronx 53, New York

<table>
<thead>
<tr>
<th>Problem</th>
<th>Traction or $\mathcal{K}^*$ eigenfunction</th>
<th>BC velocity or $\mathcal{K}$ eigenfunction</th>
<th>H.B. (1964) Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$t^{RBM}(x) = \frac{(n \cdot x) P}{4\pi abc}$</td>
<td>steady translation</td>
<td>(7.6)</td>
</tr>
<tr>
<td>(2)</td>
<td>$t^{RBM(rol)}(x) = \frac{3(n \cdot x)}{4\pi abc} (P \cdot T) \times x$</td>
<td>steady rotation</td>
<td>(7.18)</td>
</tr>
<tr>
<td>(3)</td>
<td>$t^{(E)}(x) = \frac{3}{4\pi abc} S \cdot n$</td>
<td>rate-of-strain &amp; rotation (ellipsoid is torque-free)</td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>$\psi(x) = (n \cdot x) \varphi(x)$</td>
<td>$\varphi(x)$</td>
<td>The general result (for all eigenvalues)</td>
</tr>
</tbody>
</table>

\[
< K(v), t >_w = < v, K^w(t) >_w \\
K^w_{ij}(x, \xi) = K_{ji}(\xi, x) \frac{w(\xi)}{w(x)} = K^*_{ij}(x, \xi) \frac{w(\xi)}{w(x)}
\]

\[
< v, t >_w = \int_S v \cdot t \, w dS \\
w(x) = a^{-1}(n \cdot x)
\]
Howard's authored books

Bill Showalter's 70th Birthday Tribute to Howard

University of Illinois
At Urbana-Champaign

September 7, 1999

William R. Showalter

Professor Howard Brenner
Willard Henry Dow Professor of Chemical Engineering
Massachusetts Institute of Technology
Department of Chemical Engineering
Room 66-566
Cambridge, MA 02139-4307

Dear Howie:

I believe our first meeting was in about 1959 when you presented a paper at a session I organized for a “Christmas Symposium,” then sponsored annually by the I&EC Division of ACS. If I remember correctly, Bob Pigford was in charge and the Symposium was at the University of Delaware.

But of course it was at Minnesota, probably in 1964, that we really got to know each other and each other’s families. Our shared office allowed me to ingest both your ideas and your cigar smoke. Since I still seem to be in good health, it is safe to conclude that the effects of the former outweighed those of the latter. Although it helped to share an office with you, you have profoundly influenced thousands of students and scholars through your books and papers. “Happel and Brenner” was, I believe, the catalyst stimulating enormous advances in our understanding of fluid mechanics at low Reynolds numbers.

We are all in your debt for your uncompromising standards and your clarity of expression. It is a privilege and pleasure for Jane and me to send our warmest congratulations on the occasion of your seventieth birthday. Good health and good writing in the years to come!

Fond regards,

WRS

cc: Ludwig Nitsche
Presentation for the Brenner Memorial Symposium
L. Gary Leal

My Friend
(as Fairchild Scholar at Caltech)
Howard was an inspiration to me, and his exquisite taste in problems often guided me and my students in very rewarding directions.

Howard visited me on several occasions, but the longest period of direct interactions with him came during the year that he spent as a Fairchild Scholar at Caltech.

We all learned many things from Howard during that period, and I still remember the series of lectures that he gave that was entitled something like “Getting Something for Nothing”—all about the many conclusions that could be achieved in low Re hydrodynamics from the linearity of the problem without actually having to solve any equations.

Although much of Howard’s work early work was focused on low Reynolds number hydrodynamics, another area that he worked on throughout his career was a fundamentally correct description of transport processes at fluid interfaces, and we spent his year as a Fairchild Scholar trying to develop some new ideas in this area as evidenced by the list of joint publications on this topic on the next slide.

Recently, I have been collaborating with Todd Squires in my department, on the rheology of fluid interfaces, and because of my history with Howard on this general topic, it seemed an appropriate subject for the technical part of my talk.
My Colleague and Collaborator

Howard Brenner


Measurements of Interfacial Viscosities in the Presence of Marangoni Forces

Gwynn J. Elfring*, Todd M. Squires and L. Gary Leal

Department of Chemical Engineering
University of California, Santa Barbara

*currently: Dept of Mech Eng, Univ of British Columbia
Question

Can a translating probe (shape of a disk) be used to measure the rheological properties of an interface in the presence of surfactants?

SIMPLEST CASE: A Newtonian Interface

\[ \nabla_s \Pi = \mu_s \nabla_s^2 \mathbf{u}_s + \kappa_s \nabla_s \nabla_s \cdot \mathbf{u}_s + \mathbf{f}_s \]

- The force on the probe is related to the quantities we are trying to measure, but it is modified by Marangoni effects
ANALYSIS
SIMPLEST CASE: A flat Newtonian Interface between a Newtonian bulk fluid and a gas

Analytically tractable: A thin liquid film $\delta=H/a<<1$

IDEA: Small interfacial effects, but we can still ask whether the presence of Marangoni forces compromises the ability to measure $\mu_s$ and $\kappa_s$?

Interface Momentum Eqn and Eqn of State

\[
\nabla_s \Pi = \delta Bo \left[ \nabla_s^2 u_s + \alpha \nabla_s (\nabla_s \cdot u_s) \right] - f_s \quad \text{bc. } u_s \bigg|_{r=1} = U, \quad u_s \bigg|_{r=\infty} = 0.
\]

\[
\nabla_s \Pi = \beta (E^*\nabla \ln \Gamma)
\]

$\delta Bo = (H/a)(\mu_s/\mu a); \quad \alpha = \kappa_s/\mu_s; \quad \beta = \mu_U/E_0 \delta; \quad \text{here } E_0 \text{ is } \Gamma(d\Pi/d\Gamma) \bigg|_{\Gamma=\Gamma_0}$

Bulk fluid equations of motion (solved via the thin-film (lubrication) approximation)

\[
\nabla_s p = \frac{1}{2} \nabla_s p + u_s^b; \quad \nabla_s^2 p = 6 \nabla_s \cdot u_s
\]
Surfactant conservation

\[ \nabla_s \cdot (\Gamma u_s) = \text{Pe}_s \nabla_s^2 \Gamma - \epsilon^{-1} \Delta \Gamma; \quad \text{bc. } \Gamma_{r \to \infty} = 1, \quad \epsilon_r \cdot \nabla_s \Gamma \bigg|_{r=1} = 0 \]

\[ \text{Pe}_s = Ua / D_s; \quad \epsilon = U / a k_s \]

\[ k_s = k_{on} + k_{off}; \quad \Delta \Gamma = \Gamma - 1 \text{ (scaled with } \Gamma_0) \]

**ASYMPTOTIC CASES**

3 limiting cases where the surfactant concentration tends to a constant

**Incompressible:**

\[ \beta \to 0 \quad \nabla_s \cdot u_s = 0 \]

**Compressible**

(i.e. \( \nabla_s \cdot u_s \neq 0 \))

- **Diffusive limit:** \( \text{Pe}_s \to 0 \)
- **Fast adsorption:** \( \epsilon \to 0 \)

\[ \Gamma \to \text{const.} \quad \Pi \to \text{const.} \quad \text{(first approximation)} \]
Base flow

for either $\varepsilon \ll 1$ or $Pe \ll 1$

\[ \Gamma \approx \Gamma^{(0)} \rightarrow 1 \quad \nabla \Pi \rightarrow 0 \]

\[ u_s \rightarrow u_s^{(0)} \text{ but } \nabla_s \cdot u_s^{(0)} \neq 0 \]


Force on the Disk

Bulk Dominated:

\[ \delta Bo(1 + \alpha) \rightarrow 0 \]

\[ \mathcal{R}_{FU} = \frac{2\pi \mu a^2}{H} \frac{4}{5} \]

Interface Dominated:

\[ \delta Bo(1 + \alpha) \gg 1 \]

\[ \mathcal{R}_{FU} = \frac{2\pi \mu a^2}{H} \left[ 1 - \frac{\ln(\kappa_s H/\mu a^2) - 2\gamma}{4\kappa_s H/\mu a^2} \right] \]

\[ \mu_s = 0 \]

Interface viscosities

Increase force on probe

\[ \mathcal{R}_{FU} = \frac{2\pi \mu a^2}{H} \frac{2\mu_s H/\mu a^2}{\ln(2\mu_s H/\mu a^2) - 2\gamma} \]

\[ \kappa_s = 0 \]

In general a complicated function of $\mu_s$ and $\kappa_s$ but, if know $\mu_s$ could determine $\kappa_s$
Concentration nonuniformities

• Solve for perturbation fields

\[ \Gamma(x_s) = \Gamma^{(0)} + \{ \varepsilon, Pe \} \Gamma^{(1)}(x_s) + ... \]
\[ u_s(x_s) = u_s^{(0)}(x_s) + \{ \varepsilon, Pe \} u_s^{(1)} + ... \]

Surfactant conservation eqn

\[ \nabla_s \cdot (\Gamma u_s) = Pe_s \nabla^2 \Gamma - \varepsilon^{-1} (\Gamma - \Gamma_0) \]

\[ \nabla^2 \Gamma^{(1)} = \nabla_s \cdot u_s^{(0)} \]
\[ \Gamma^{(1)} = -\nabla_s \cdot u_s^{(0)} \]

• Marangoni forces increase resistance

\[ \mathcal{R}_{FU} = \mathcal{R}_{FU}^{(0)} + Ma \mathcal{R}_{FU}^{(1)} \quad \text{where} \quad Ma = \{ \varepsilon, Pe \} \beta^{-1} \]

So, the divergence (or dilatation) of the base flow leads to surfactant concentration variations and these, in turn, lead to Marangoni flows; both \( u_s^{(0)} \) and \( u_s^{(1)} \) depend on \( \mu_s \) and \( \kappa_s \), and thus so too do the corresponding contributions to the force.
Small Péclet

Marangoni number

\[ Ma = Pe \beta^{-1} = \frac{HE_0}{\mu D} \]

Bulk Dominated: \( \delta Bo (1 + \alpha) \to 0 \)

\[ R_{FU}^{(1)} = \frac{2\pi \mu a^2}{H} \frac{2}{25} \quad \Rightarrow \quad R_{FU} = R_{FU}^{(0)} \left( 1 + \frac{Ma}{10} \right) \]

(independent of surface viscosities)

Interface Dominated: \( \delta Bo (1 + \alpha) \gg 1 \)

\[ R_{FU}^{(1)} = \frac{2\pi \mu a^2}{H} \frac{1}{16\kappa_s H/\mu a^2} \quad \mu_s = 0 \]

increase of dilatational viscosity decreases Marangoni force

\[ R_{FU}^{(1)} = \frac{2\pi \mu a^2}{H} \frac{\mu_s H/\mu a^2}{4 (\ln(2\mu_s H/\mu a^2) - 2\gamma)^2} \quad \kappa_s = 0 \]

increase of shear viscosity increases Marangoni force
Force

\[ 
\mathbf{F} = -e_x \left[ F^{(0)} + \text{Ma}F^{(1)} \right] \quad F^{(n)} = R_{FU}^{(n)} U 
\]
Fast adsorption/desorption
(This limit is singular and requires a boundary layer at the surface of the disk)

Marangoni number

\[ \text{Ma} = \epsilon \beta^{-1} = \frac{E_0 H}{k_s \mu a^2} \]

Interface Dominated:

\[ \delta Bo (1 + \alpha) \gg 1 \]

\[ R_{FU}^{(1)} = \frac{2 \pi \mu a^2}{H} \frac{\ln(\kappa_s H/\mu a^2) - 2 \gamma - 1}{(2 \kappa_s H/\mu a^2)^2} \quad \mu_s = 0 \]

increase of dilatational viscosity decreases Marangoni force

\[ R_{FU}^{(1)} = \frac{2 \pi \mu a^2}{H} \frac{\ln(\mu_s H/\mu a^2) - 2 \gamma - 1}{(\ln(2 \mu_s H/\mu a^2) - 2 \gamma)^2} \quad \kappa_s = 0 \]

increase of shear viscosity decreases Marangoni force
**Force**

\[ \mathbf{F} = -e_x \left[ F^{(0)} + Ma F^{(1)} \right] \quad F^{(n)} = R^{(n)}_{FU} U \]

\[ \mu_s = \kappa_s \]

Interpretation of measured force in terms of interface viscosities is difficult
Conclusions

• Probe translation generally gives rise to mixed-type flows with a dilatational component.

• Dilatation leads to Marangoni forces that depend nontrivially on interface viscosities (and relaxation timescales at interface).

• Translational resistance was derived for thin films, assuming small deviations from equilibrium, and the result is a complicated dependence of the Marangoni force on the probe on the surface viscosities.

• We conclude that the translating disk-shaped probe is a suboptimal interface rheometer.
It is heartwarming to observe the solemn care, and professional and personal warmth, with which John Anderson and Sangtae Kim organized both the memorial session in honor of Howard Brenner at the 2014 AIChE Annual Meeting, and the present volume collecting the presentations made at that time. As former students of this profoundly wise and kind quintessential theoretician, we feel honored and grateful for the opportunity to record a few of our thoughts about him. Aside from his prodigious scientific output per se, his major impact as a mentor upon several generations of researchers is evident in the academic family tree presented at the session, reproduced in this volume.

Howard Brenner tackled problems in low Reynolds number hydrodynamics and other areas of transport phenomena that were both important and challenging—mathematically, physically, and conceptually. His trademark attribute was rigor, and he invariably rooted out the final and definitively correct answers to problems involving hindered diffusion through pores, transport of flexible molecules, effective transport properties of microstructured and composite media, and other processes. It is worthwhile to comment specifically upon one aspect of his scientific method that was constantly on display in the illustrations for his articles and the transparencies he used in giving seminars and presentations. Conspicuous in these vibrant (and legendary) works of art was the use of well-defined, well-chosen idealized models of molecules, macromolecules, and particles, and of the microscopic structures in which they move. Thus, many of his papers addressed, for example, the motion of spheres and ellipsoids near flat walls or in circular cylindrical pores. This focus was a wise one, because invaluable insight issues from a rigorous and detailed analysis of an idealized system that captures the essential features of a given physical situation. Such an approach yields the main mechanistic explanation of an observed transport phenomenon, and also identifies its
underlying mathematical structure in generality. Molecular-level details (for example, the precise shapes of a basically round molecule and a basically flat nearby wall) may produce some modest numerical deviations from the hydrodynamic properties resulting from a sphere-plane calculation, but the idealized model truly explains what is happening. Properly calibrated, the idealized model also usually turns out to be quite accurate quantitatively. By always settling with rigor the general problem using a well-chosen idealized prototype of the physical reality, Brenner derived results of universal and lasting value. This point is elaborated upon further below in the context of biophysics.

Howard Brenner addressed intrinsically interesting physical questions, and he was virtuosic in handling the complicated mathematics that arose in settling these questions. He was definitely a theoretician, and appreciated the esthetic side of his chosen enterprise, as has been remarked for example by David Edwards. Nevertheless, one must marvel at the instincts that invariably steered him toward problems that could not have been more useful or relevant to applications. In this respect he was a man years or even decades ahead of his time, a point which arose in conversation and correspondence with Leslie Lipschultz, one of Brenner’s three daughters. Getting to know her in the wake of her father’s passing has been a profoundly positively experience. Any reasonable assessment of Brenner’s direct impact on applications would certainly have to include four areas that are very topical today, namely: (i) biophysics and biomedical engineering; (ii) microfluidics; (iii) nanotechnology; and (iv) multiscale modeling. The elapse of time and the advancement of technology and computation brought these fields to find—and need—Brenner’s classic works.

Regarding items (i) and (ii), biophysicists at least as early as the 1950s and 60s recognized that creeping flow fluid mechanics was directly applicable to the biological membranes and tissues studied by them. The reason is that the small (molecular and macromolecular) scale they worked in automatically makes for small values of the Reynolds number. They were avid users of any available hydrodynamic formulas for spheres (as models for small molecules, proteins, and ions) moving in Stokes flow near walls and in pores. This is exactly the type of problem that Brenner worked on. Although intrinsically interesting from the purely mathematical perspective, it was also directly relevant to physiological applications. Therefore, hydrodynamic results for low-Reynolds number particulate flows were never purely esoteric. Indeed, speaking in broad terms, one might say that Brenner’s papers fulfilled the wish list of needed theoretical results constantly on the mind of biophysicists and theoretical biologists for decades. Shortly after Brenner’s passing one of us entertained a visit from a graduate student interested in modeling cellular motion near the active wall of a diagnostic flow cell. In typical fashion (“typical” because such incidents occur periodically), the conversation revolved around several of Brenner’s classic papers on motion of spheres in quiescent fluid and in shear flow above a planar wall (Chem. Eng. Sci. 16:242–251 (1961); 22:637–651 (1967); 22:653–660 (1967)). It was a poignant reminder of his continuing impact.

Regarding item (iii), nanoparticles are also so tiny that the Reynolds numbers characterizing their motions are exceedingly small. Although nanodots and other nanoparticles are spherical, the growing ability to fabricate exotic shapes (nanorods, etc.) represents an area where Brenner’s extensive work on nonspherical shapes comes directly into play.

Regarding item (iv), Brenner’s work on macrotransport processes established a rigorous and general approach to averaging over details of a material that has some kind of structure at a small scale
to characterize it in terms of average properties observable at a larger scale. The process can be carried out several times, for example first going from the molecular scale to a microscopic scale, and then again from the microscopic scale to a macroscopic scale. It is often now referred to as “multiscale modeling,” and advances are being made in its computational implementation in many areas. One such area is modeling of drug and chemical diffusion through skin, which the two of us among many other investigators are involved in. What is now called multiscale modeling is an area that Brenner developed much of the framework for, and radically expanded the scope of, starting around the late 1970s. A good early example of its application in the dermal context is the four-scale theoretical tour de force by Edwards and Langer (J. Pharm. Sci. 83:1315–1334 (1994)).

The preceding examples show that Howard Brenner’s work has always been extremely applicable to practical problems. It is the supreme compliment that can be given to a theoretician. We do not even touch the areas of interfacial dynamics, particulate separations, and rheology of suspensions of particles and bubbles, for which many further examples could be given, because others would be far more qualified to provide commentary in these areas.

Sometimes theoretical imagination can be hindered by the difficulty of mathematical problems that would have to be solved in applying general theory to specific cases. Brenner was intellectually fearless: supposing we could solve a certain set of problems in principle, he asked, what could we do with that knowledge? It was in this way that he forged ahead in enormous conceptuality generality—years before computation brought the mathematical problems within reach.

Also arising in correspondence with Leslie Lipschultz was Brenner’s last work, on volume transport and bivelocity fluid mechanics. As remarked by one of us, “A nineteen-page, single-author paper, using force of reason to challenge conventional ideas on an important question in continuum mechanics, impeccable in prose, filled with difficult physics and mathematics that your father could handle like few others, is quintessential Howard Brenner.” As recounted by Lipschultz, he finished this paper in his last days, breathing with the help of an oxygen bottle, when moving across his study was an enormous physical exertion. It is also very much worth remarking that Ronald Probstein was a sterling friend to Brenner, so ably shepherding this final manuscript through to its posthumous publication (Phys. Rev. E 89:043020 (2014)).

In the final analysis it is difficult to say which is the more profound of the two types of Brenner’s impact—his prodigious output of scientific papers and three monographs (including the legendary Low Reynolds Number Hydrodynamics), or his mentoring of graduate students and postdocs. We feel truly fortunate and infinitely grateful to have been his Ph.D. students, and to have existed in the awe-inspiring milieu — of fluid mechanics, macrotransport, mathematics, deep conversations, and the dual quest for rigor and generality—that surrounded him. Better or more inspiring graduate training does not exist. Ultimately the highest praise we can give is that—young and inexperienced though we were—he truly respected us, giving us the benefit of his wisdom and what we needed to hear, but also always reminding us that we ourselves were the determinants of our fate. It is actually unnecessary to choose between scientific publications or mentoring, because Brenner excelled at both to a singular degree. One hopes ardently that he knew how deeply he was appreciated as both a superlative scientist and a fine human being.
Remarks on Howard Brenner from Two Grateful Students—Johnnes M. Nitsche and Ludwig C. Nitsche

Howard Brenner's M.S. & Ph.D. Students

Irvin Pliskin (M.Ch.E., 1962, NYU)
Louis Theodore (Eng.Sc.D., 1964, NYU)
Arthur J. Goldman (Ph.D., 1966, NYU)
Richard M. Sonshine (Ph.D., 1966, NYU)
Guili A. Feldman (Ph.D., 1967, NYU)
David L. Ripps (Ph.D., 1966, NYU)
Peter M. Bungay (Ph.D., 1970, CMU)
Barry R. Hirschfield (Ph.D., 1972, NYU)
Lawrence J. Gaydos (Ph.D., 1977, CMU)
Mordecai Zuzovsky (Ph.D., 1976, CMU)
Loren H. Dill (Ph.D., 1981, U. of Rochester)
John C. Kunesh (Ph.D., 1971, CMU)
Eugene J. Cone (Ph.D., 1972, CMU)
Shi-Woo Rhee (Ph.D., 1984, MIT)
Ali Nadim (Ph.D., 1986, MIT)
Stephanie Dungan (M.S., 1988 MIT)
David A. Edwards (Ph.D., 1987, MIT; co-advised with D.T. Wasan IIT)
Johannes M. Nitsche (Ph.D., 1980 MIT)
Ludwig C. Nitsche (Ph.D., 1989 MIT)
Gretchen M. Mavrovouniotis (Ph.D., 1989 MIT)
David C. Guell (Ph.D., 1991 MIT)
Michael T. Kezirian (M.S., 1992, MIT)
Arthur Lue (M.S., 1992, MIT)
James R. Abbott (Ph.D., 1993, MIT)
Alejandro E. Mendoza-Blanco (Ph.D., 1996, MIT; degree from UNAM)
Richard P. Batczyk (Ph.D., 1995, MIT)
Moshe (Marc) Van Dyke (M.S., 1995, MIT)
Siqian He (Ph.D., 1996, MIT)
Michelle D. Bryden (Ph.D., 1997, MIT)
Venkat Ganesan (Ph.D., 1999, MIT)
Kevin D. Dorfman (Ph.D., 2002, MIT)
Carlos M. Rinaldi (Ph.D., 2002, MIT)
James R. Bielenberg (Ph.D., 2004, MIT)
Aruna Mohan (Ph.D., 2007, MIT; co-advised with Patrick S. Doyle)

David A. Edwards, Harvard U.
Rachel Yeld (2012)

Ludwig C. Nitsche, UIC
Shan Zhan (M.S., 1993)
Weidong Zhang (Ph.D., 2004)
Tajah Shah (M.S., 2005; co-advisor)
Olga Jedry (M.S., 2005)
Javier Rios (M.S., 2006)
Prashanth Parthasarathi (Ph.D., 2008)

Johannes M. Nitsche, U. at Buffalo
Satpute P. Nanagudi (M.S., 1993)
Sreenivas Golapudi (M.S., 2004)
Ying-Ta Wu (Ph.D., 1994)
Partha Roy (Ph.D., 1996)
Jeevan Baliga (Ph.D., 1996)
Xin L. Lindquist (M.S., 1993)
Maichen Chang (Ph.D., 2003)
Tiwei Wang (Ph.D., 2003)
Xiaomai Xie (Ph.D., 2004)
Ankit Verma (Ph.D., 2007)
Yuri Dancik (Ph.D., 2007)

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Bharadwaj Narayanan (Ph.D., 2006)
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Megha M. Surve (Ph.D., 2006)
Jamie M. Xropka (Ph.D., 2008)
William Kredberg (Ph.D., 2009, co-advisor)
Manas R. Shah (Ph.D., 2009)
Y. Limdri Xhoxhakolli (Ph.D., 2010)
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Kevin D. Dorfman, U. of NY
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Nortonli Arabi (M.S., 2010)
Nabil Zaqzouq (Ph.D., 2010)
Scott Xing (Ph.D., 2014)
Margaret Einuk (Ph.D., 2012)
Daniel Olom (Ph.D., 2012)
Douglas Tree (Ph.D., 2014)
Jose Thomas (Ph.D., 2014)
Srirama Datta (Ph.D., 2014)

Howard Brenner's postdoctoral associates

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Christopher C. Reed
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H. Saun Selleor
David van der Merve
Paul C.H. Chan
Michele Vignes-Adler
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Genady ('Gil') Joselvskii
Peter M. Bungay
Shimon Haber
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Michael Shapiro
Francesco Mancini
Juan Camacho
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During the years 1975-1978, I enjoyed a number of dinners with Howard. Invariably the conversation turned to some technical topic. Howard was clearly trying to expand his research from pure hydrodynamics into other areas of physics. My PhD work on interparticle forces appealed to him as one possible new direction. Now, having the benefit of 40 years of experience, the topic which I believe Howard was searching for in 1975 is electrokinetic phenomena.

The research problem I will describe today is one example. A uniform electric field externally applied around a charged sphere near a wall exerts a body force on charged fluid elements next to the sphere, thereby driving flow of the fluid which entrains nearby particles. In an alternating field, the net entrainment can either bring nearby particles together to send them apart, depending unexpectedly on which salt is dissolved in the water. This unexpected result would have appealed to Howard.
I first met Howard when I interviewed for a faculty position at Carnegie Mellon in 1974. In my dissertation research concerning the effect of colloidal forces on transport of nanoparticles, I had made use of several of his publications dealing with mobility of particles near walls. The fact that the normal mobility vanishes at contact with the wall had a profound effect on the rate of deposition.

With the luxury of two other offers for a faculty position, my decision to come to CMU was heavily influenced by the reputation of the department and especially by Howard’s personal appeal. I was very flattered that someone with his reputation would be interested in me.

Once I started teaching at CMU in January of 1975, Howard took me under his wing and served as my mentor. I was particularly sorry to see him leave in 1978 to become department head at Rochester.
I choose this research topic for today’s talk because I thought you would find the mystery interesting and because one of Howard’s papers provides the key to unravelling the mystery. So let me start by presenting the mystery.

You are looking at 6-micron polystyrene latex particles dispersed in a dilute aqueous solution about 1 mm thick, sandwiched between two large, transparent parallel-plate electrodes. Using a microscope, we are looking down through the transparent upper electrode at the particles which have settled near the lower electrode. They are not stuck to the electrode, but are levitated above the electrode by double-layer repulsion.

Before the electric field is turned on, some Brownian motion of the particles can be seen. After the electric field is turned on (1.8 kV/m at 100 Hz), the particles jump together in a second or two.
This second video is taken under exactly the same conditions, except the electrolyte has been changed: KCl has been replaced by KOH. But once the electric field starts, the particles move apart rather than aggregate – completely opposite behavior compared with previous video.

One obvious difference between KCl and KOH is that KOH is a strong base, but the solution is very dilute so the pH of KOH is about 10 whereas the pH of KCl is about 6. This difference in pH does not significantly alter the charge on either the particles or on the electrodes. Yet the response of the particles is completely opposite for the two electrolytes. This is the mystery.

The first question to answer is “why do the particles move at all?” Since the particles are charged, you expect them to feel a force in the direction of the electric field – but that electric field is normal to the screen in the videos you just saw – so the motion we are seeing is orthogonal to the electric field.
The answer to this question was provided by our session chair – John Anderson – and his group who studied the response of the particle to d/c electric fields (instead of a/c). This schematic shows a side view rather than the top view shown in the videos. John showed that the action of the electric field on the charged counterion cloud surrounding the particle (indicated by the orange halo) generates electroosmotic flow up over the surface of the particle. This flow leaves the north pole of the sphere and returns to the sphere along the surface of the electrode, thus generating the closed streamlines shown in this slide.

Any other particles nearby can become entrained in this flow and are dragged toward the target particle. This is how d/c fields cause aggregation of particles.
Of course, if the direction of the electric field is reversed, the direction of flow in the streamlines will be reversed. Thus we expect that adjacent particles would then move apart rather than together. So d/c fields can either cause aggregation or disaggregation, depending on the polarity of the electric field.

In a/c fields, the electric field acts upward for half of each cycle and downward for the other half. Thus we would expect that particles will move apart for half of each cycle and then move together for the second half. This would lead to no net lateral motion. Yet in the videos, you saw significant lateral motion which persisted over 1000’s of cycles.

For any net lateral motion in a/c fields to occur, some break in the symmetry of the two half-cycles is needed.
Near the planar electrode, the electroosmotic flow can be approximated by linear shear flow. This flow entrains nearby particles at a speed which depends on the elevation $h$ of the particle: the higher the elevation, the faster the entrainment speed of the particle. Here is where Howard solved the mystery for us: Howard calculated the entrainment speed $U$ of the particle in linear shear flow. Far from the wall, the particle is entrained at the same speed as the undisturbed fluid at its center. Near the wall, the particle lags behind the fluid.

If the average elevation is higher during one half of the cycle compared to that of the other half, that difference in the two averages will lead to a net drift either toward aggregation or disaggregation, depending on which half has the higher elevation. This serves to break the symmetry of the two halves of the cycle.
Changes in elevation are too small to be detected using video microscopy. Instead we used total internal reflection microscopy (TIRM) to monitor the elevation of isolated single particles over time. Brownian motion obscures the deterministic changes in elevation arising from the electric field.

We averaged the response over 100’s of cycles to average out the stochastic fluctuations, leaving the deterministic contribution shown as the red curve in this slide. Notice that the elevation varies between 120 and 220 nm. These elevations are only a few percent of the radii of the particles. Thus Howard’s correction for wall effects is really needed.

Also shown as the blue curve is the electric current passing between the electrodes. According to Ohm’s law, the current is proportional to the electric field with the fluid conductivity serving as the proportionality constant. Thus the blue curve also represents the instantaneous electric field acting on the particle.

Notice that the maximum in the elevation occurs at a later time than the maximum in the electric field which is driving the changes in elevation. We can measure this delay time and convert it into a phase angle. It turns out that a phase angle of 90 degrees corresponds to equal average elevations for the two halves of the cycle. In this particular experiment, the phase angle was 81 degrees.
Phase angles larger than 90 degrees correspond to the average elevation being larger for the half of the cycle corresponding to aggregation; thus we observe net aggregation behavior for the 4 experiments shown in the right graph, which plots center-to-center distance versus time for an isolated pair of nearby particles.
This slide summarizes 5 different experiments in which the phase angles turn out to be less than 90 degrees. Isolated particle pairs disaggregate during these experiments.

In each of the experiments on the last two slides we observed a perfect correlation between the aggregation-disaggregation response and whether the phase angle was greater than or less than 90 degrees. For his dissertation, James Hoggard used the measured phase angle to predict the rate of change in center-to-center distance within a particle pair and obtained reasonable agreement with the rates measured.
But James could not predict the phase angle for a given electrolyte; he had to use the measured phase angle. The relationship between phase angle and electrolyte choice was provided by Chris Wirth (now at Cleveland State). It turns out to the ion mobilities affect the dynamics of the particle’s motion.

This slide shows a different phase angle from ours: this is the phase between the velocity of a particle far from any boundary and the electric field. Even in bulk electrophoresis, the ion mobilities are important to the electro-convective-diffusion of ions in the counterion cloud surrounding the particle. Here you can see that a difference of a couple of degrees in phase angle arises at 100 Hz from a change in electrolyte.

Chris Wirth extended this analysis to predict the dynamic response of particles near to the electrode, which is considerably more complicated.
Conclusions

- Lateral motion of particles arises from their entrainment in electroosmotic flow
- Break in symmetry arises from different average elevation during two halves of cycle
- Electrolyte choice matters because ion mobilities affect dynamics

I am saddened by the realization that I will never again experience Howard’s sparkling wit while attending an AIChE meeting. But at the same time, I am delighted to add my voice to those of the other speakers in this session honoring the brilliance of his research and his skill in mentoring others who do research.
I am grateful to the organizers of this session for providing me with an opportunity to recount some of the interactions I’ve had with Howard Brenner over a period of more than 50 years, and to appreciate once more how much I’ve been enriched by that experience. I also wish to express my appreciation to Joe Goddard, who did the legwork of assembling a bibliography of Howard’s publications. Evidently Howard had the quaint opinion that his papers were more valuable than lists attesting to their number.

I believe we first met at a meeting of the Industrial and Engineering Chemistry Division of the American Chemical Society, then known as the annual “Christmas Symposium” and held in 1961 at the University of Delaware. I chaired the Symposium that year, and Howard presented a paper. We had the chance to discuss each other’s work, but it was at the University of Minnesota in 1964 that we came into daily contact for several months. Howard and I were on sabbatical leaves from our home institutions (New York University and Princeton, respectively), and the Department of Chemical Engineering at Minnesota was considered by many, and certainly by us, to be the department most densely populated with young academics eager to apply newly found principles of applied mathematics to the foundational challenges of chemical engineering.

As office-mates in a stimulating environment, we soon learned what motivated us, and I quickly gained an enduring admiration for Howard’s depth as a scholar, his wry sense of humor, and his balance of life’s priorities. The classic Low Reynolds Number Hydrodynamics by Happel and Brenner was soon to appear, and Howard lectured on the subject to a collection of graduate students and faculty through the spring quarter of 1964. I have jealously guarded my notes from that course because they continue to be the most lucid exposition of the subject I have ever encountered. As I pore over the pages a half-century later, I’m amazed that he worked out numerous
derivations in great detail, displayed superb blackboard technique with figures and equations, and managed to retain our attention and interest through the intricacies of spherical harmonic expansions and singular perturbations; all the more impressive because my recollection is that we met for 1-1/2 hours at a sitting. I believe Howard’s success was due to his mental engagement as he went through endless derivations, rather than simply copying a set of notes from paper to board. Occasional bits of humor also helped. Because he was fond of referring to the “arbitrary body” around which there was a low-Reynolds number flow, he invented the word “Arbitron” for the ultimate arbitrary body. (Little did Howard know that in the same era a company formed to measure audience size of radio stations was actually named “ Arbitron.”)

Most of us can identify one or more mentors who were crucial to our professional development. For Howard, such a person was John Happel (see Fig. 1), already a distinguished chemical engineer with deep industrial experience who had become chairman of NYU’s chemical engineering department. I recall Howard talking about his own applied education at Pratt Institute and a project involving enhancement of the cleaning power of some brand of silver polish. Howard was John’s star student and was asked to join the NYU faculty even before he finished his Ph.D. there. Howard told me that Happel had an ambition to write three books, each representing one of the widely different areas of his expertise, and he did. (See Fig. 2.)

<table>
<thead>
<tr>
<th>Three Books Authored or Coauthored by John Happel</th>
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<tbody>
<tr>
<td><strong>Chemical Process Economics</strong> John Wiley, 1958</td>
</tr>
<tr>
<td><strong>Low Reynolds Number Hydrodynamics</strong> Prentice-Hall, 1965</td>
</tr>
<tr>
<td><strong>Isotopic Assessment of Heterogeneous Catalysis</strong> Academic Press, 1986</td>
</tr>
</tbody>
</table>

It is not unusual for a mentor to do such a good job that the mentee wishes to move at a faster and less traditional pace than the mentor desires. I believe this happened with Howard and John. Thus, when CMU offered attractions NYU was not prepared to provide, Howard moved to Pittsburgh and spent 11 productive years there.

Whether the impressions come indirectly from reading Howard’s papers, or directly, as they did for me during our shared time at Minnesota, it becomes clear that Howard was driven by two overriding goals: clarity and generality. I will demonstrate these qualities, as well as the evolution of Howard’s thinking, by describing a remarkable series of five single-author papers, all published in *Chemical Engineering Science* between 1963 and 1966. All had the same title, “The Stokes Resistance of an Arbitrary
Particle,” with the subtitles shown in Fig. 3. The series shows his penchant for generality through a quest for complete knowledge of movement of an arbitrary [rigid] body surrounded by a fluid moving at vanishingly small Reynolds number. It is significant that of the 26 papers Howard published between 1961 and 1967, the word “arbitrary” appears 17 times in the titles; once it is used twice in the same title. Another favorite word in that era was “intrinsic.” I eventually stopped counting the number of times I encountered it.

“The Stokes Resistance of an Arbitrary Particle”
Howard Brenner

*Chemical Engineering Science, 1963-1966*

(I.) 1963. **18**, 1-25
II. “An extension” 1964. **19**, 599-629
III. “Shear fields” 1964. **19**, 631-651

Before addressing the technical advances contained in these papers, a few remarks about their style are relevant to the evolution of Howard’s thinking and to his persona. First, there is no subtitle to the first paper, which is why I have enclosed the Roman (I) in parentheses. I believe that when Howard titled this paper, he intended it to be the full story on his subject. Second, the series, as well as the lecture notes mentioned earlier, show his fascination with Gibbs’ polyadic notation. Howard was a master on the blackboard with multiple underscores and dots. Third, his choice of references is one more indication of his love for generality and clarity. He was fond of primary sources. In Paper (I) there are 31 references. Nine of these were published prior to 1935. They include papers by Einstein and Gibbs, and they include the foundations of the subject. Finally, Howard is generous in his acknowledgements. In each paper he expresses gratitude to people who provided useful comments. One of these for Paper (I) was his early mentor, John Happel. In marked contrast to today’s papers, with the exception of Paper V there are no references to sources of funding.

Now let us review the content of the first two papers of this remarkable series. Howard considered a rigid body of arbitrary shape moving in translation at velocity $\mathbf{U}$ through an incompressible Newtonian fluid which, in the absence of the body, is at rest. (See Fig. 4.) In the absence of body forces, fluid motion is governed by

$$\nabla \cdot \mathbf{v} = 0 \quad (1)$$

$$\nabla p = \mu \nabla^2 \mathbf{v} \quad (2)$$

where $\mathbf{v}$ is the fluid velocity, $p$ the pressure, and $\mu$ the viscosity. The stress tensor $\Pi$ is given by

$$\Pi = -p \mathbf{I} + \mu (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) \quad (3)$$

where $\mathbf{I}$ is the unit tensor (called, not surprisingly, the *idem* factor by Brenner).

Figure 3. The series of five papers published in *Chemical Engineering Science, 1963-1966.*

Figure 4. A rigid “arbitrary” body translating at velocity $\mathbf{U}$ through an infinite expanse of fluid otherwise at rest.
An important additional tool for the analysis is the reciprocal theorem, derived in the paper by Brenner for two motions of two incompressible fluids in the Stokes regime, with the two situations designated by ‘′’ and ‘′,” respectively.

\[ \mu^\prime \nabla \cdot ([\nabla \cdot v]^\prime) = \mu^\prime \nabla \cdot ([\nabla \cdot v]^\prime) \]  (4)

With these ingredients Brenner showed that the force \( F \) exerted by the fluid on the body of Fig. 4, i.e.,

\[ F = \int \!\! s p \cdot d S \cdot [\nabla \cdot v] \]  (5)

can be reduced to

\[ F = - \mu K \cdot U \]  (6)

where \( K \) is a symmetric second-order tensor “intrinsic” to the body, and hence it contains three orthogonal eigenvectors such that if \( U \) is aligned with one of them, the force and velocity will be collinear. A similar argument leads to an analogous expression for the torque about a point \( o \),

\[ T_o = \int \!\! s p \cdot \int d S \cdot [\nabla \cdot v] \]  (7)

Thus for a body rotating with angular velocity \( \omega \),

\[ T_o = -\mu \Omega_o \cdot \omega \]  (8)

where \( \Omega_o \) is also symmetric and intrinsic to the body but, of course, is dependent on the location of point \( o \).

With these results Brenner further identified a point \( r_o \), the “center of hydrodynamic stress,” which, analogous to the relation of the center of mass to the force of gravitation, is a point about which the moment of hydrodynamic force on the body is zero. He summarizes the accomplishments of Paper (I) as follows:

“These two relations [force and torque], in conjunction with the concept of a centre of hydrodynamic stress, permit a complete calculation of the motion of an arbitrary body in an infinite medium through application of Newton's laws of motion.”

But there was more—again illustrating the drive for generality. The acknowledgement in Paper II demonstrates that Howard’s mentor was still influencing him and had opened the door to further generality. Howard writes,

“I am deeply grateful to Professor John Happel for originally suggesting that propeller-like bodies might spin as they settled, and for prodding me [sic] to the point of carrying out a crucial experiment along these lines.”

Paper II makes clear that particle geometries exist for which one cannot disentangle rotation and translation, and locate a center of hydrodynamic stress, as implied in eqns. (6) and (8). Instead, Brenner uncovered an additional tensor, the coupling tensor, \( C_o \). Then eqns. (6) and (8) become, respectively,

\[ F = -\mu (K \cdot U_o + C_o^T \cdot \omega) \]  (9)

and

\[ T_o = -\mu (C_o \cdot U_o + \Omega_o \cdot \omega) \]  (10)

The coupling tensor is also intrinsic to the body, but in general it is not symmetric, and it depends on the position \( o \) about which the torque and the translational velocity are measured. Brenner
shows that any body possesses a unique point \( R \) for which \( C_0 \) is symmetric. Thus \( R \), which he calls the center of reaction, has the same significance as the center of hydrodynamic stress of Paper (I).

He further describes the geometric characteristics of some examples for which the coupling tensor is not zero. These are bodies with a type of skew symmetry, such as propellers. One can find an example in nature, where certain seeds, such as those of the box elder tree (Fig. 5), will fall to the ground with a spinning motion generated by the force of gravity. (The process does not take place at “zero” Reynolds number, but the qualitative effect is represented by a non-zero coupling tensor.)

The remaining three papers of the series continue to increase the generality begun in Paper (I), providing further evidence of Brenner’s drive for generality in what he did.

Howard left us with a remarkable store of knowledge of low Reynolds number hydrodynamics. But he also left us with an enduring example of personal characteristics that mark a scholar of great distinction. In the hope his example will be a point of reference for generations to come, I wish to present the Brenner recipe for such distinction, and I cite six characteristics. They are:

1. Pick good problems
2. Learn what is relevant from prior work
3. Work hard and with intellectual honesty
4. Find important new insights
5. Share your thoughts with colleagues
6. Retain a sense of balance and good humor

Howard received many honors acknowledging his scholarly accomplishments. One of them was an honorary degree conferred by Clarkson University in 2004. In his acceptance remarks is a brief paragraph that explains why those of us who knew him well, know how fortunate we were; and those who didn’t, know how much they missed:

“In high school I had a difficult time in geometry. But I had a wonderful teacher, Mr. Woods, who did a theorem proving that the interior angles of a triangle added up to 180 degrees. This was a purely theoretical exercise, one that I had never thought about before. It was hard to believe that you could use your own mind to predict something and then go into the world and measure it. I had a protractor because I was taking a course in mechanical engineering, so after class I drew some triangles and they added up to 180 degrees. I was amazed. And I have been amazed ever since.”

Figure 5. Seeds of the box elder tree.
Top row, left to right: A young Howard with his parents Margaret and Max, and sister Renee; a Brooklyn Tech "bad" report card—and a "good" report card; a mea culpa—running in the lab at Brooklyn Tech.

Bottom row, left to right: Revelry in a line dance with friends; first business card; at daughter Leslie's wedding, 1974
Top row, left to right: Howard with daughters and first grandchild Margo, 1978; Howard alone with Margo, 1978; at daughter Joyce’s wedding, 1981; at daughter Suzanne’s wedding, 1997

Middle row, left to right: With granddaughter Margo at her bat mitzvah, 1991; at Margo’s bat mitzvah, 1991; with wife, Lorraine, and children, 1991; at grandson Daniel’s Bar Mitzvah in Israel with wife, Lisa, 1999

Bottom row: With grandson Alex in Illinois; daughter Leslie’s Ode to her dad, the professor
Top row, left to right: Family gathering on a cruise to Canada, 2005; Howard and a friend at a gathering with Howard’s grandchildren, 2011; two professors—Howard with daughter Suzanne

Middle row, left to right: With daughters on a cruise to Canada, 2005; at grandson Max’s bar mitzvah, 2011; with daughter Leslie in New York City

Bottom row: Howard and Lorraine with daughter Leslie, 1950s
ACKNOWLEDGMENTS

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John L. Anderson
Illinois Institute of Technology
March 1, 2016
On November 16, 2014, the American Institute of Chemical Engineers sponsored a technical symposium in honor of Howard Brenner’s work and life. This book is based on those presentations. Many of us felt that a more permanent remembrance of Howard should be created—something to which future engineers and scientists could refer to learn more about the man.