Mohr’s Circle

Academic Resource Center
Introduction

• The transformation equations for plane stress can be represented in graphical form by a plot known as Mohr’s Circle.

• This graphical representation is extremely useful because it enables you to visualize the relationships between the normal and shear stresses acting on various inclined planes at a point in a stressed body.

• Using Mohr’s Circle you can also calculate principal stresses, maximum shear stresses and stresses on inclined planes.
Stress Transformation Equations

\[ x_1 \frac{x + y}{2} = \frac{x}{2} \cos 2 \theta + xy \sin 2 \theta \]  

\[ x_1y_1 \frac{x + y}{2} = \frac{x}{2} \sin 2 \theta + xy \cos 2 \theta \]
Derivation of Mohr’s Circle

• If we vary $\theta$ from $0^\circ$ to $360^\circ$, we will get all possible values of $\sigma_{x_1}$ and $\tau_{x_1y_1}$ for a given stress state.

• Eliminate $\theta$ by squaring both sides of 1 and 2 equation and adding the two equations together.

$$\left(\sigma_{x_1} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x_1y_1}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$
Derivation of Mohr’s Circle (cont’d)

Define $\sigma_{avg}$ and $R$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substitute for $\sigma_{avg}$ and $R$ to get

$$\left(\sigma_x - \sigma_{avg}\right)^2 + \tau_{x1y1}^2 = R^2$$

which is the equation for a circle with centre $(\sigma_{avg},0)$ and radius $R$. 
Mohr’s Circle Equation

• The circle with that equation is called a Mohr’s Circle, named after the German Civil Engineer Otto Mohr. He also developed the graphical technique for drawing the circle in 1882.

$$\left(\sigma_{x1} - \sigma_{avg}\right)^2 + \tau_{x1y1}^2 = R^2$$

• The graphical method is a simple & clear approach to an otherwise complicated analysis.
Sign Convention for Mohr’s Circle

- Shear Stress is plotted as positive downward
- $\theta$ on the stress element $= 2\theta$ in Mohr’s circle

\[
(\sigma_{x1} - \sigma_{\text{avg}})^2 + \tau_{x1y1}^2 = R^2
\]
Constructing Mohr’s Circle:
Procedure
1. Draw a set of coordinate axes with $\sigma x_1$ as positive to the right and $\tau x_1 y_1$ as positive downward.
2. Locate point $A$, representing the stress conditions on the $x$ face of the element by plotting its coordinates $\sigma x_1 = \sigma_x$ and $\tau x_1 y_1 = \tau_{xy}$. Note that point $A$ on the circle corresponds to $\theta = 0^\circ$.
3. Locate point $B$, representing the stress conditions on the $y$ face of the element by plotting its coordinates $\sigma x_1 = \sigma_y$ and $\tau x_1 y_1 = -\tau_{xy}$. Note that point $B$ on the circle corresponds to $\theta = 90^\circ$. 
4. Draw a line from point A to point B, a diameter of the circle passing through point c (center of circle). Points A and B are at opposite ends of the diameter (and therefore 180° apart on the circle).

5. Using point c as the center, draw Mohr’s circle through points A and B. This circle has radius $R$. The center of the circle c at the point having coordinates $\sigma x_1 = \sigma_{\text{avg}}$ and $\tau x_1 y_1 = 0$. 
Stress Transformation: Graphical Illustration
Explanation

• On Mohr’s circle, point A corresponds to $\theta = 0$. Thus it’s the reference point from which angles are measured.

• The angle $2\theta$ locates the point $D$ on the circle, which has coordinates $\sigma x_1$ and $\tau x_1 y_1$. $D$ represents the stresses on the $x_1$ face of the inclined element.

• Point $E$, which is diametrically opposite point $D$ is located $180^\circ$ from $cD$. Thus point $E$ gives the stress on the $y_1$ face of the inclined element.

• Thus, as we rotate the $x_1y_1$ axes counterclockwise by an angle $\theta$, the point on Mohr’s circle corresponding to the $x_1$ face moves ccw by an angle of $2\theta$.
Principal Stresses

- **B (θ=90)**
- **A (θ=0)**
- **2θ_p1**
- **2θ_p2**

Diagram showing the principal stresses and strain angles for a given configuration.
**Explanation**

- Principle stresses are stresses that act on a principle surface. This surface has no shear force components (that means $\tau_{x_1 y_1} = 0$).
- This can be easily done by rotating A and B to the $\sigma_{x_1}$ axis.
- $\sigma_1$ = stress on $x_1$ surface, $\sigma_2$ = stress on $y_1$ surface.
- The object in reality has to be rotated at an angle $\theta_p$ to experience no shear stress.
Maximum Shear Stress

Note carefully the directions of the shear forces.
Explanation

• The same method to calculate principle stresses is used to find maximum shear stress.

• Points A and B are rotated to the point of maximum $\tau_{x_1y_1}$ value. This is the maximum shear stress value $\tau_{\text{max}}$.

• Uniform planar stress ($\sigma_s$) and shear stress ($\tau_{\text{max}}$) will be experienced by both $x_1$ and $y_1$ surfaces.

• The object in reality has to be rotated at an angle $\theta_s$ to experience maximum shear stress.
Example 1

Draw the Mohr’s Circle of the stress element shown below. Determine the principle stresses and the maximum shear stresses.

What we know:
\[ \sigma_x = -80 \text{ MPa} \]
\[ \sigma_y = +50 \text{ MPa} \]
\[ \tau_{xy} = 25 \text{ MPa} \]

Coordinates of Points
A: \((-80,25)\)
B: \((50,-25)\)
Example 1 (cont’d)

\[ c = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-80 + 50}{2} = -15 \]

\[ R = \sqrt{(50 - (-15))^2 + (25)^2} \]

\[ R = \sqrt{65^2 + 25^2} = 69.6 \]
Example 1 (cont’d)

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\[ \sigma_{1,2} = c \pm R \]
\[ \sigma_{1,2} = -15 \pm 69.6 \]
\[ \sigma_1 = 54.6 \text{ MPa} \]
\[ \sigma_2 = -84.6 \text{ MPa} \]
Example 1 (cont’d)

\[ \tau_{\text{max}} = R = 69.6 \text{ MPa} \]
\[ \sigma_s = c = -15 \text{ MPa} \]
Example 2

Given the same stress element (shown below), find the stress components when it is inclined at 30° clockwise. Draw the corresponding stress elements.

What we know:
\[ \sigma_x = -80 \text{ MPa} \]
\[ \sigma_y = +50 \text{ MPa} \]
\[ \tau_{xy} = 25 \text{ MPa} \]

Coordinates of Points:
A: (-80,25)
B: (50,-25)
Example 2 (cont’d)

Using stress transformation equation ($\theta=30^\circ$):

\[
x_1 = \frac{x + y}{2} = \frac{x}{2} \cos^2 \theta + xy \sin^2 \theta
\]

\[
x_1y_1 = \frac{x + y}{2} \sin^2 \theta + xy \cos^2 \theta
\]

\[
\sigma_x = -25.8 \text{ MPa} \quad \sigma_y = -4.15 \text{ MPa} \quad \tau_{xy} = 68.8 \text{ MPa}
\]
Example 2 (cont’d)

Graphical approach using Mohr’s Circle (and trigonometry)