Physics Workshop: Conservation of Linear Momentum

ARC Staff Workshop Sessions
Introduction

• How do we analyze a body moving when a force is acting on it? We use Newton’s Second Law. If there are forces that are easy to examine, then we can use the basic principles to figure out how an object will move.

• What do we do if we have situations when we have very complex forces.

    Ex: A tennis player hits a ball with a racket. How do we get around the complex forces associated with the deformation of both bodies?

Here we can employ the powerful idea of conservation of Linear Momentum.
What is this Principle About?

• This principle says that when the net external forces on a system are zero, then the total linear momentum of the system remains the same.
• An intuitive example is this:
• If a stationary firecracker were to explode into two parts, then the individual pieces would move opposite to one another. This keeps the total momentum of the system zero.
  • Mathematically,
    • $\sum_{k=1}^{n} m_{k,i}v_{k,i} = \sum_{i=1}^{n} m_{k,f}v_{k,f}$
    • “The sum of the momenta before and after is the same”
Requirements to apply this principle

• The bodies under consideration must be isolated in a system such that there is no net external force acting on the system.
• Think of this as “putting all your bodies in a box which has no net external forces acting on it.
• Notice the word ‘NET’. This is very important. The force on a body need not be zero, but the total sum of forces must be zero.
How to set up a problem?

• A picture always helps; draw a picture.
• Isolate the bodies to study.
• Pick two points in time, one initial and one final.
• Isolate the bodies in a system with no net external forces.
• Find initial and final momenta expressions for each object. Use subscripts ‘i’ and ‘f’ for initial and final conditions. Use subscripts ‘1’ and ‘2’ for bodies.
• Sum up initial momenta and final momenta.
• Equate initial and final momenta to each other.
• Solve for variable/s.
Important tip!!

• When examining one dimensional motion, the equations are easier to solve as algebraic equations.
• If the motion is two dimensional, then the use of vector equations considerably simplify calculations. Also, in this case, imposing a co-ordinate axis also helps.
Example Problem

• A 4.0 kg puck sliding on a frictionless surface explodes into two 2.0 kg bodies, one moving at 3.0 m/s due north, and the other 5.0 m/s 30 degrees north of east. Find the initial velocity of the puck?

• This is what a picture would look like:
Example Problem (contd.)

• You can see that if you put the two final bodies in a system along with the earth, then there is no net external force on them.

\[
\begin{align*}
\text{V1} &= 5.0 \text{m/s} \\
\text{V2} &= 5.0 \text{m/s}
\end{align*}
\]
Example problem (contd.)

• $V_i = ?$; 
  (The variable to be solved)

• $v_{1,f} = 5.0 \cos(30)i + 5.0 \sin(30)j$; 
  (Vector equation for velocity of $v_{1,f}$)

• $v_{1,f} = 3.0 \cos(90)i + 3.0 \sin(30)j$; 
  (Vector equation for velocity of $v_{2,f}$)

• $p_{1,f} = 2.0[5.0 \cos(30)i + 5.0 \sin(30)j]$; 
  (p=momentum=mv)

• $p_{2,f} = 2.0[3.0 \cos(90)i + 3.0 \sin(30)j]$;

• $P_i = 4.0V_i$;

• Equate initial and final momenta:
  • $P_i = p_{1,f} + p_{2,f}$
  • $4.0V_i = 2.0[5.0 \cos(30)i + 5.0 \sin(30)j] + 2.0[3.0 \cos(90)i + 3.0 \sin(30)j]$
Example problem (contd.)

• Solving for $V_i$,

\[ V_i = \frac{2.0[5.0 \cos(30)i + 5.0 \sin(30)j] + 2.0[3.0 \cos(90)i + 3.0 \sin(30)j]}{4.0} \]

• This yields,

\[ V_i = \frac{10.0\sqrt{3}i + 5.0j + 6.0j}{4.0} \]

\[ V_i = 5 \frac{\sqrt{3}}{4} i + \frac{11.0}{4.0} j \]

\[ V_i = (2.2i + 2.8j) \text{ m/s} \quad (\text{to two significant figures}) \]
References and for further problems

• Fundamentals of Physics, Halliday and Resnick (2004)