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Understanding Concepts

Kinetic Energy

\[ K = \frac{1}{2}mv^2 \]
Kinetic Energy

The 3rd Edition Ohanian Physics books defines kinetic energy thusly:

“A body in motion has energy of motion, or kinetic energy.”
Potential Energy

According to Ohanian, potential energy is “the capacity of a particle to do work by virtue of its position in space.”

There are many kinds of potential energy, but the ones we will be focusing on are gravitational and elastic potential energy.
Understanding Concepts

Gravitational Potential Energy

\[ U = mgy \]

Elastic Potential Energy

\[ U = \frac{1}{2}kx^2 \]
Understanding Concepts

Conservation of Energy

\[ E = U + K = \text{[constant]} \]
Example Conceptual Situations

Potential Energy High
Kinetic Energy Low

Potential Energy Low
Kinetic Energy High
Example Conceptual Situations

\[ E = K + U \]
\[ E = 0 + U \]
\[ E = U \]
\[ E = mgh \]

\[ E = K + U \]
\[ E = K + 0 \]
\[ E = K \]
\[ E = \frac{1}{2}mv^2 \]
Example Conceptual Situations

For now:
\[ E = K + U \]
\[ E = U \]
\[ E = \frac{1}{2}kx^2 \]

Once the bowstring returns to equilibrium:
\[ E = K + U \]
\[ E = K \]
\[ E = \frac{1}{2}mv^2 \]
Example Problems

Chapter 7 #69:

A wrecking ball of mass 600 kg hangs form a crane by a cable of length 10 m. If this wrecking ball is released from an angle of 35 degrees, what will be its kinetic energy when it swings through the lowest point of the arc?
Example Problems

Solution:

The first step is to find the composition of the initial and final energies. In this case, the initial energy is the energy of the wrecking ball before it is released at 35 degrees. At this time, the ball has no velocity, and therefore the kinetic energy is also zero, so $E = U$. As there are no springs involved in this problem, the potential energy is going to be completely gravitational.

On the other hand, when the ball is at the lowest point in its swing, it can be considered to be at zero height, and therefore it will have no gravitational potential energy.
Example Problems

Solution:

\[ E_i = \text{Initial Energy (at 35 degrees)} \]
\[ E_f = \text{Final Energy (at the bottom of the swing)} \]
\[ E_i = U + K \ (v = 0) \]
\[ E_f = U + K \ (h = 0) \]
\[ E_i = U + 0 \]
\[ E_f = 0 + K \]
\[ E_i = mgh \]
\[ E_f = \frac{1}{2}mv^2 \]

\[ E_i = E_f \]

\[ mgh = K \]
Example Problems

Solution:

Now that we have the kinetic energy at the bottom of the swing in terms of the potential energy at the beginning, we just need $h$, and we can find $K$.

As we can see, the height that the ball starts at is $L - L \cdot \cos(35)$, where $L$ is the length of the cable.

If we plug this into the equation for $K$, we get $K = m \cdot g \cdot L(1 - \cos(35))$

Plugging in the values gives roughly 11kJ for the kinetic energy.
Example Problems

Chapter 7 #73:

A jet aircraft looping the loop flies along a vertical circle of diameter 1000 m with a speed of 620 km/h at the bottom of the circle and a speed of 350 km/h at the top of the circle. The change of speed is due mainly to the downward pull of gravity. For the given speed at the bottom of the circle, what speed would you expect at top of the circle if the thrust of the aircraft’s engine exactly balances the friction force of air?
Example Problems

Solution:

The first step is to find the composition of the initial and final energies. In this case, the initial energy is the energy of the jet before it starts the loop. At this time, the jet is traveling at 620 km/h, at it is at the lowest point of the loops so $h = 0$ which means $E = K$. As there are no springs involved in this problem, the potential energy is going to be completely gravitational.

On the other hand, when the jet is at the top point in its loop, it still has some velocity, but the majority of it has transferred as kinetic energy into potential energy, as the height of the plane has increased, so $E = U + K$. 
Example Problems

Solution:

\[ E_i = U + K \quad (h = 0) \quad \quad E_f = U + K \quad (v_f < v_i) \]
\[ E_i = 0 + K \quad \quad E_f = U + K \]
\[ E_i = \frac{1}{2}mv_i^2 \quad \quad E_f = mgh + \frac{1}{2}mv_f^2 \]
\[ E_i = E_f \]

\[ \frac{1}{2}mv_i^2 = mgh + \frac{1}{2}mv_f^2 \]

\[ h = 1000 \text{ m} \quad \mid \quad v_i = 620 \text{ km/h} = 172.2 \text{ m/s} \quad \mid \quad v_f = ? \]

Solving for \( v_f \) gives \( v_f = 100 \text{ m/s} \).
Example Problems

Chapter 8 #11:

A bow may be regarded mathematically as a spring. The archer stretches this “spring” and then suddenly releases it so that the bowstring pushes against the arrow. Suppose that when the archer stretches the “spring” 0.52 m, he must exert a force of 160 N to hold the arrow in this position. If he now releases the arrow, what will be the speed of the arrow when the “spring” reaches its equilibrium position? The mass of the arrow is 0.020 kg. Pretend that the “spring” is massless.
Solution:

The first step is to find the composition of the initial and final energies. In this case, the entire problem takes place horizontally and therefore the gravitational potential energy is constant at all times and can be ignored. The initial energy (when the bow is drawn all the way back) is elastic potential energy, and the kinetic energy is zero as the system is at rest. $E = U$.

On the other hand, once released, the bow converts this initial energy into kinetic energy as the string nears its equilibrium position. At the point of equilibrium for the string, the energy is completely transferred to kinetic energy. $E = K$. 
Example Problems

Solution:

\[ E_i = U + K \ (v = 0) \]
\[ E_i = U + 0 \]
\[ E_i = \frac{1}{2}kx^2 \]

\[ E_f = U + K \ (x = 0) \]
\[ E_f = 0 + K \]
\[ E_f = \frac{1}{2}mv^2 \]

\[ E_i = E_f \]

\[ \frac{1}{2}kx^2 = \frac{1}{2}mv^2 \]
\[ k = \frac{F}{x} \ (\text{Hooke's Law}) \]
\[ \frac{1}{2}(F/x)x^2 = \frac{1}{2}mv^2 \]

\[ m = 0.02 \text{ kg} \mid F = 160 \text{ N} \mid x = .52 \text{ m} \]

\[ v = 64.5 \text{ m/s} \]
Example Problems

Chapter 8 #19 part a:

Mountain climbers use nylon safety rope whose elasticity plays an important role in cushioning the sharp jerk if a climber falls and is suddenly stopped by the rope. Suppose that a climber of 80 kg attached to a 10 m rope falls freely from a height of 10 m above to a height of 10 m below the point at which the rope is anchored to a vertical wall of rock. Treating the rope as a spring with $k = 4900 \text{ N/m}$ calculate the maximum force that the rope exerts on the climber during stopping.
Example Problems

Solution:

The first step is to find the composition of the initial and final energies. In this case, the initial energy is the energy climber before he falls. At this time, the rope is not stretched out, and he is at rest, so \( E = U \) where \( U \) is gravitational potential energy.

On the other hand, our final energy is going to be at the most stretched point of the rope (as \( F = kx \) will give the largest value for the largest \( x \).) At this point, which we will treat as the lowest point the climber reaches (\( h = 0 \)), the climber is once again at rest, so \( E = U \), where \( U \) is elastic potential energy.
Example Problems

Solution:

\[
\begin{align*}
E_i &= U + K \ (v = 0, \ x = 0) \\
E_f &= U + K \ (h = 0, \ v = 0) \\
E_i &= U + 0 \\
E_f &= U + 0 \\
E_i &= mgh \\
E_f &= \frac{1}{2}kx^2 \\
E_i &= E_f \\
mgh &= \frac{1}{2}kx^2 \\
h &= 20 + x \\
m^*g(20 + x) &= \frac{1}{2}kx^2 \\
m &= 80 \text{ kg} \ | \ k = 4900 \text{ N/m} \ | \ g = 9.81 \text{ m/s}^2 \\
\text{Use the quadratic formula to solve for } x. \\
x &= 2.69 \text{ m} \\
\text{Hooke’s Law:} \\
F &= kx = 13 \text{ kN}
\end{align*}
\]